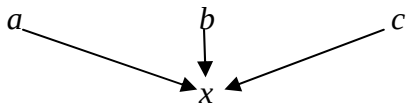
Acyclic Directed Graph of Spinoza's *Ethics*

This paper is a scouting expedition. My aim is to present two tools and demonstrate a few of their uses. I hope to show just enough to suggest that the tools can lead to a new slant on Spinoza's *Ethics*, and that they delineate classes of propositions within the book that belong together and illuminate one another.

The tools I describe here are not entirely new. Lancelot R. Fletcher, for example, provides a table closely related to the adjacency table. (It can be found on the web at http://frank.mtsu.edu/~rbombard/RB/Spinoza/ethica .) Nevertheless, I believe that commentators have not pressed these tools as far as they deserve, and I will take them one step further.

Consider a large set of proofs, each with this structure: "From *a*, *b*, and *c*, we derive *x*." Taken together, these proofs constitute a directed graph. This proof (for instance), might be represented:



If a further statement were derived from *x* (and perhaps from other statements), then an arrow would lead out of *x*, and on to that statement. Suppose further that we have a constraint that forbids "circular reasoning." That is, the graph cannot contain any cycles: routes that lead (in the direction of the arrows) out of a statement and eventually back into it. Then we have an "acyclic directed graph." Spinoza's *Ethics* is such an acyclic directed graph.

Considering *Ethics* as a graph leads naturally to certain questions, and to grouping together statements in the work that play similar roles in the graph. To explain these groupings, I need to remind the reader of some notions from graph theory.

Nodes and Edges. In the graph we are discussing, a statement is called a "node", and the arrow leading from one statement to another is called an "edge."

Connectedness. A graph is "connected" if every node can be reached along some edge (going with, or against, the arrows).

In-degree and Out-degree. The In-degree of a node is the number of edges leading into it. Its Out-degree is the number of edges leading out of it. In the example above, *a*, *b*, and *c* have in-degree = 0 and out-degree = 1; *x* has in-degree = 3 and out-degree = 0.

Path. A "path" is a series of alternating nodes and edges through a graph (in the direction of the arrows) from one node to another.

Let us begin here and ask a few questions about *Ethics*.

Is the graph connected?

First, we should notice that *Ethics* begins with assumptions—Spinoza calls them axioms, postulates, and definitions. These are nodes with In-degree = 0: they are taken without proof. Second, the work is acyclic and finitely long, so there are statements that are not used to prove anything else—later, I will call these "leaves." These statements have Out-degree = 0. But if the graph is connected, then there would be no nodes with both In- and Out-degree = 0.

Ethics is not connected. It contains 42 nodes that cannot be reached, forward or backward, on any path. We would ordinarily think of these statements as "unused assumptions." They are assumed without proof, and then not used to prove anything else. This is an interesting class. Twenty-six of them are definitions of emotions (for example, indignation, drunkenness, and others), and eleven are definitions of such things as duration, individual things, knowledge of various kinds, the definition of *natura naturans* and *natura naturata*. The unused definitions of the emotions are really "leaves by accident": doubtless Spinoza had nothing to say about them that he had not already made clear about other emotions—other than to show that they could be understood within his system.[1]

The remaining five are 1A02 (Not conceived through another, through itself.); 2O02 (Some body parts are fluid, some soft, some hard); 2A08 (Changability of bodies depends on adjoining surfaces); 3O02 (The human body is capable of suffering many changes and retaining traces of objects); and 5A02 (The power of an emotion is limited by the power of its cause…). These unused assumptions are concerned with the intended interpretation of the system: what kind of thing we are to think of as modes.

How many nodes and edges are there in the graph?

To some small extent, the answer to this question depends on the policies used to extract the graph from the text. Here are the policies I use. First, a node is a statement that Spinoza numbers. (So the "Observations" in the Appendix to Part 4 are not nodes.) Second, only when Spinoza explicitly states that a node derives from another node and gives its number does it count as an edge. Often, in a proof, Spinoza will quote or paraphrase an earlier statement without citing its number; I do not count that as an edge because I wish to make my errors easily detectable and my results repeatable. Hence, I have adopted the narrowest reasonable test for the presence of an edge. My third policy is that if a proposition has two or more proofs, then its immediate predecessors include the immediate predecessors in all of the proofs.

This is also an appropriate place to explain my abbreviations for Spinoza's statements and how to parse them. Each statement has a name on this pattern: the first character is the "Part" of *Ethics*. The second character is 'A' for axiom, 'D' for definition, 'E' for definition of an emotion, 'O' for postulate, or 'P' for proposition. The next two characters are the position within that class of statements. Thus 1A01 is "Axiom 1 of

Part I"; 2P34 is "Proposition 34 of Part II". A corollary gets a 'C' and two digits following the abbreviation of its principal: so 5P40C01 is the first corollary of Proposition 40 of Part 5.[2] Besides making the abbreviations human readable, this numbering system mirrors the acyclic character of the graph: a statement *a* is not available for use in proving *b*, unless its abbreviation precedes *b's* in alphabetic order. (Notice that 'A,', 'D', 'E' and 'O' precede 'P' in alphabetic order, so within a "part" such statements appear before that part's proved propositions. Axioms, definitions, and postulates are not proved at all, so they need only precede the part's propositions. According to the naming convention, "propositions" are placed in the same order as they appear in *Ethics*, and corollaries come immediately after their principals.)

With these preliminaries out of the way, *Ethics* has 430 nodes and 1035 edges. Of the nodes, 107 have In-degree = 0—the axioms, definitions, and postulates, including the unused assumptions, and also including 3P04 that Spinoza says is "self-evident". 103 have Out-degree = 0 (not including the unused assumptions)—the "leaves". Thus there are 323 proved propositions.

Which statements are most used in proofs?

You may remember, from your days in geometry class, the three propositions called "Side-angle-side", "Angle-side-angle" and "Side-side-side." They were three ways of establishing the congruence of triangles, and were used in almost every proof. In graph-theory terms they had very large Out-degree. This class of statement is interesting because they show the most useful and characteristic features of the object Spinoza is describing. There are three sub-groups of these hard-working statements. I list them, with their Out-degrees.

Group I

1P15	17	Whatever is, is in God, and nothing can either be or be conceived without God
1P16	14	From the necessity of the divine nature, infinite numbers of things in infinite ways (that is to say, all things which can be conceived by the infinite intellect) must follow.
2P11C01	15	The human mind is a part of the infinite intellect of God
3P03	19	The actions of the mind arise from adequate ideas alone, but the passive states depend upon those alone which are inadequate
2P40	13	Those ideas are also adequate which follow in the mind from ideas which are adequate in it.

Group II

2P07	14	The order and connection of ideas is the same as the order and connection of things
2P11	11	The first thing which forms the actual being of the human mind is nothing else than the idea of an individual thing actually existing.
2P13	14	The object of the idea constituting the human mind is a body, or a certain mode of extension actually existing, and nothing else.
2P16	13	The idea of every way in which human body is affected involves the nature of the body and external body
2P17	17	If the human body is affected in a way which involves the nature of an external body, the human mind will contemplate that body as existing or as present, until the human body be affected by a modification which excludes the existence or presence of the external body.
3P11	27	If anything increases, diminished, helps, or limits our body's power of action, the idea of that thing increases, diminishes, helps, or limits our mind's power of thought.

Group III

3P07	21	The effort by which each thing endeavors to preserve its own being is nothing but the actual essence of the thing itself.
3P13	25	Whenever the mind imagines those things which lessen or limit the body's power of action, it endeavors as much as possible to recollect what excludes the existence of those things.
4P26	11	All efforts which we make through reason are nothing but efforts to understand, and the mind, in so far as it uses reason, adjudges nothing as profitable to itself except that which conduces to understanding.
4P27	7	We do not know that anything is certainly good or evil except that which conduces to understanding, or which can prevent us from understanding.

4P37 12 The good which everyone who follows after virtue seeks, for himself, he will desire for other men; and his desire on their behalf will be greater in proportion as he has a greater knowledge of God.

Read together, these propositions have a marked coherence, and they even seem to form the skeleton on which the *Ethics* hangs. On the other hand, some of the most famous propositions are missing—for instance, the proposition that there is only one substance. It is easy to explain this phenomenon: these are the working propositions of the system…the ones which facilitate the transition from one proposition to another. As the body's distinctive features are not all found in its skeleton, so the distinctive outlines of Spinoza's system are not all found here at its core.

The grouping that I have introduced is meant to highlight some features of these skeletal principles. Group I consists of a number of closure principles; Group II contains the isomorphism of the attributes; the third group are a peculiar kind of structural principle in ethics, which, though rather common, seems not to have an accepted name. Let us examine each in turn.

Closure principles of Group I connect the membership of a set to a relation or operation. For instance, we might define the Alden family as consisting of just John Alden and anyone who is a child of someone who is already in the family. Or, we might think of Spinoza's system as consisting of the axioms and immediate successors of statements already in the system. When we are thinking of the family or the system after this manner, we speak of them as "closed under 'child of'" or "closed under 'immediate successor'".

Sometimes, also, a closure principle assures us that an operation can be performed repeatedly without trespassing the boundaries of a set that is known in some other way. For instance, it is a closure principle that assures us that the sum of two positive integers is another positive integer, rather than (for instance) a pot of geraniums; and it is a failure of closure when the difference between two positive integers is not necessarily a positive integer.

In *Ethics*, closure principles have several important roles to play. One such principle assures us that the causes and effects of natural states are themselves natural states, and that therefore forbids miracles. Indeed, the effects of extended things are themselves extended things, and the effects of ideas are ideas. Another closure principle assures us that the logical outcomes of adequate ideas are themselves adequate—giving us a foundation for the deductive development of science.

Commentators who are attempting to understand Spinoza's notions of substance, attribute and mode should perhaps give closer attention to two facts. First, the attributes partition the modes. That is, every mode belongs to exactly one attribute. Second, attributes are causally closed, and no set of modes short of the attribute is causally closed. If each attribute is causally closed, the union of the attributes will be causally closed as well. Hence, the substance is also causally closed, and we may think of each attribute as

expressing this important feature of substance. Moreover, an attribute is the *least* item that expresses the substance: individual bodies, for instance, although they are complex structure of modes, are not causally closed.

> All the modes by which one body is affected by another follow from the nature of the body affected, and at the same time from the nature of the affecting body, so that one and the same body may be moved in different ways according to the diversity of the nature of the moving bodies, and, on the other hand, different bodies may be moved in different ways by one and the same body. (2A07)

The suggestion that we treat Spinozistic attributes as the least classes of modes closed under causality might seem a bit far-fetched and anachronistic. However, it is quite clear that Leibniz's monads are specified in just this way. And, (although here the argument is a bit more tenuous) Descartes' substances (each of which has exactly one attribute) seem also to be entities which are causally closed and which have no causally closed parts. Although the vocabulary of causal closure is anachronistic, it names a way of thinking explicit in Leibniz and prefigured in Descartes—so it would not be surprising to find Spinoza thinking that way as well.

Why are the **isomorphism principles** in Group II of the workhorses necessary? Let us assume, for the moment and for the sake of argument, that each attribute is a lattice: that is, within the attribute, any pair of modes has a least upper bound and a greatest lower bound with respect to causality. (So, for any pair of extended objects, they have a common remote cause and a common remote effect.) Now, assuming that each attribute is a lattice, is the "substance"—the union of the attributes—a lattice? The answer is "No." The substance is causally closed (since the cause of any mode that is in the substance is also in the substance), but the substance is not a lattice, since modes of different attributes have no causes or effects in common.

If we follow out the line of thought that has been sketched in the preceding paragraph, it will be clear why isomorphism principles (of the kind found in Group II) are so important in Spinoza's proofs. If there are many causally closed attributes, the question must arise whether they "belong together" in some sense, or whether our metaphysics is just pluralistic and that is that. Spinoza's answer is that each attribute exhibits the same structure as every other attribute—and thus, that it is appropriate to say that they are different aspects of one divine nature.

Once more, the parallel with Leibniz is instructive. Faced with a cloud of causally closed monads, Leibniz restores unity to his worldview by postulating that there is a pre-established harmony among the internal structures of the monads. The pre-established harmony, of course, is the most conspicuous sign of God's presence in the world. But while Spinoza insists that the attributes must have the *same* structure, Leibniz is content to postulate a similarity relation and to settle for the consequence that there is a single world that each monad reflects from its own point of view.

To display an isomorphism—a 'Sameness of structure'—we need to specify several things: first, the two sets of objects that are the domains of the structures; then, a mapping between the two domains; and then, whatever operations and relations are to be mirrored. Spinoza is clear enough (I believe) about the domains: extended objects and adequate ideas. He is clear enough about the mapping: each object maps to "its" idea. He is much less clear about the operations and relations to be preserved under the mapping. Clearly, the causal structure is to be preserved, so that if an object *A* causes *B*, then the idea of *B* must follow from the idea of *A*. It also appears that both ideas and things are subject to "composition", and that composition is mirrored between attributes. Thus if the human body has a kidney and a liver, then the idea of the body must involve the ideas of a kidney and of a liver. Finally, it appears that modes have internal states, and that they interact in such a way that their inner state is changed. If *A* and *B* are pre-existing modes, and they interact, the internal state of *A* will reflect both its prior state and the state of *B*, and similarly for *B*.

Composition and the way in which modes affect one another appear to interact in a complex way. The clearest representation is in the Note to Lemma 7 of part 2:

> We thus see in what manner a composite individual can be affected in many ways and yet retain its nature. Up to this point, we have conceived an individual to be composed merely of bodies which are distinguished from one another solely by motion and rest, speed and slowness, that is to say, to be composed of the most simple bodies. If we now consider an individual of another kind, composed of many individuals of diverse natures, we shall discover that it may be affected in many other ways, its nature nevertheless being preserved. For since each of its parts is composed of a number of bodies, each part (by the preceding lemma), without any change of its nature, can move more slowly or more quickly, and consequently can communicate its motion more quickly or more slowly to the rest. If we now imagine a third kind of individual composed of these of the second kind, we shall discover that it can be affected in many other ways without any change of form. Thus, if we advance *ad infinitum*, we may easily conceive the whole of nature to be one individual whose parts, that is to say, all bodies differ in infinite ways without any change of the whole individual. If it had been my object to consider specially the question of a body, I should have had to explain and demonstrate these things more fully. But, as I have already said, I have another end in view, and I have noticed them only because I can easily deduce from those things which I have proposed to demonstrate.

Since we are on a scouting expedition, this is probably as far as we can usefully go in exploring the isomorphism of the attributes. At the same time, we must recognize that the issues are there to be explored. For instance, Spinoza seems to recognize that there

are difficult problems concerning true ideas about non-existent objects. In the explicitly ethical parts of the book, he relies heavily on what we might call second-level ideas that are reflections on our state of mind. These second-level ideas are hard to map back to states of the attribute of extension. We will have a brief discussion of this issue in a few moments.

This mirroring of logical consequence and causality is the key to many of Spinoza's characteristically ethical positions. In a rough way, the more composite an object (idea or extended object) is, the nearer it is to being self-determining…that is, to have the cause of its doings within itself. If we imagine composition extending so far that all the modes of an attribute can be seen as a single mode, then that mode would have nothing "outside" itself that could shape it. At every level of composition short of this, the state of each mode depends on what is inside it and what is inside other modes of the same attribute. It follows from the isomorphism, then, that a mind holding the largest and most composite ideas will be unshakable, since the course of its thought will be determined entirely by its own nature. Such a mind will, metaphorically, hold the extended world within itself.

Spinoza adds an interesting twist to this mirroring early in Part III of *Ethics*. The first two parts of *Ethics* may be thought of as "statics". These two parts could be interpreted as referring to an unchanging hierarchy of intelligences (within the attribute of thought) and a more and more inclusive space corresponding to this hierarchy in the attribute of extension. (I do not mean to say that Spinoza has only this interpretation in mind: only that little in the first two parts precludes this interpretation.) At the beginning of Part III, he develops two ideas that were first introduced early in Part II: we learn that an extended object can grow and shrink and change the modes of which it is composed without becoming a different mode, and that minds can be now larger, now smaller. In addition, minds contain ideas not only of their internal state, but also ideas of their *changes* of internal state.

In a way, these additional, second-level, ideas can be thought of as the "first derivative" of the mind's state with respect to time. If my mind's internal state on Monday consists (say) of Newtonian physics, and on Tuesday I come to understand Special Relativity, then I will also have an idea of the direction and magnitude of the change in my ideas. As Spinoza writes in the note to 3P11,

> We thus see that the mind can suffer great changes, and can pass now to a greater and now to a lesser perfection; these passive states explaining to us the emotions of joy and sorrow. By "joy," therefore, in what follows, I shall understand the passive states through which the mind passes to a greater perfection; by "sorrow," on the other hand, the passive states through which it passes to a less perfection.

This notion has a great deal to do with how we think about joy, sorrow, and desire. These emotions are neither something that happens to us nor something that we do: they are representations of the direction and rate of a change. That change may be brought about in us either passively or actively. If we can understand the mechanisms that bring

about a larger or a smaller mind, we will, at the same time, understand what will give us joy or sorrow. Earlier, we noted that the single hardest-working proposition in *Ethics* is 3P11: "If anything increases, helps, or limits our body's power of action, the idea of that thing increases, diminishes, helps, or limits our mind's power of thought." This proposition (and its converse, 5P01), give Spinoza's peculiar slant to the rest of *Ethics*: joy is not something to be sought, but a measure of the degree to which self-determination is attained.

Let us turn, finally and briefly, to the workhorses that I have called **Group III**, above. It was argued long ago, by Höffding, that Spinoza's metaphysics is novel and exciting, while his ethical position is conventional and traditional. While this is perhaps true—I do not dispute the point—nevertheless, the structure of Spinoza's argument throws an interesting light on one possible foundation of the (perhaps traditional) ethical view he holds.

Many ethical views are developed under the guidance of meta-ethical symmetry principles that demand that any satisfactory ethics should be equally justified when looked at from different points of view. For instance, Kant insists that a moral law should be one that we can accept whether we consider ourselves its legislator or its subject. In a Utilitarian ethics, the calculation of advantages must be carried out in such a way that every participant would arrive at the same judgment as to the most desirable action. Rawls explores the idea that principles of justice would be chosen under certain conditions of equality and in ignorance of the respective roles of the participants. And so on.

We have already seen that Spinoza's metaphysical system imposes a very strict isomorphism upon the attributes of thought and extension. "The Good"—whatever it may turn out to be—will therefore have to look the same, whether it is viewed from the perspective of mentality or of the action of one body on another. The causal closure of the two attributes, moreover, means that we cannot say that the role of the mind is to serve bodily needs; and on the other hand, it would be an absurd violation of closure to regard mortification of the flesh as the key to spiritual beatitude. Each attribute has to be granted its full integrity: whatever is good would have to seem so, even if the universe were all mind and even if it were all body.

The meta-ethical constraints imposed by Spinoza's metaphysics lead to some very specific observations regarding the good. Activities in pursuit of wealth, power, and control of others cannot pass the test of being viewed from their mental side. Activities that lead to self-aggrandizement or to the development of private, narrow, or secret beliefs and practices appear just silly when seen under the aspect of the mental life—where an idea's adequacy is measured in part by its compass. And on the other hand, since the pursuit of joyless knowledge cannot be the end of contemplation, it is clear that action must lead to an ever-deepening participation in and understanding of the world as a whole.

The propositions of Group III are central to the development of Spinoza's ethical views because of the way they allow the meta-ethical constraints of the system to be brought to bear on proposed ethical principles. They play (approximately) the role of Kant's different formulations of the categorical imperative in bringing symmetry conditions to bear on ethical matters.

What can we learn from the leaves?

Leaves, as you will remember, are nodes with Out-degree = 0. If the previous groups of statements were means to developing the graph, the leaves are sought as ends in themselves. A statement can become a leaf in several ways. First, there are the unused assumptions (which we have discussed earlier). Second, a statement can be a corollary of another statement—low-hanging fruit, as it were. Third, it may occur so late in the graph that Spinoza put down his pen before drawing any conclusions from it. And finally, it may be of a special kind that I will call a "supporting observation."

The supporting observations are a very special group. There are perhaps twenty of them. I will take one, 3P47 ("The joy which arises from our imagining that what we hate has been destroyed or has been injured is not unaccompanied with some sorrow.") as an example.

Think for a moment about the way in which the Copernican theory of planetary motion was confirmed. Each night, the superior planets--Mars, Jupiter, and Saturn--move a bit eastward against the background of the stars. About once a year, these planets stop moving eastward, back up to the westward, and then resume their eastward march. This retrograde motion of the superior planets had been known for centuries, and models of the universe since Greek times had dealt with it; but it had always seemed an oddity. The Copernicans pointed out a rarely-noticed fact: the maximum retrograde motion always occurs when the planet crosses the observer's meridian at midnight. Now the geometry of the Copernican solar system means that when the planet transits at midnight, the earth is exactly between the planet and the sun, and so the retrograde motion is an appearance, caused by the earth, on its inside track, overtaking the superior planet and passing it. What had been an oddity in older systems became a *prediction* in Copernican astronomy.

Similarly, we can think of Spinoza's supporting observations like this: "Wise men (and even men who are much less than wise) have noticed certain things about our feelings and reactions to circumstance—for instance, that when we see our enemies come to grief, we have a moment's regret mixed with our general delight. But (once we have learned how to understand ourselves), we see that this not just chance, but the *inevitable* result of human nature."

Returning to Höffding's observation that Spinoza's ethics is not original, we may say in response that Spinoza's effort is to help us understand the way we feel, and to see that a range of feelings, emotions, and actions must occur, and must be as they are, because of the larger structure of nature.

Paths

One common question about *Ethics* is whether a given statement is derived from another. To answer this question, we could trace all the in-bound paths (not just the in-bound edges) into the given statement to see whether any includes the target. But if we mean to answer a number of these questions, then it is worth our while to create the graph's "transitive closure."

The transitive closure of a graph, *A*, is another graph, *T*, having the same set of nodes, but a larger class of edges. In particular, if there is a path from *a* to *x* in *A*, then there is an edge, <*a*,*x*> in *T*. The name, "transitive closure" comes about in this way: the edges of a graph form a binary relation; and a binary relation is called "transitive" if, whenever *aRb* and *bRc*, then also *aRc*. That is, if you can get from *a* to *c* in two steps, then you can get there in one. (And, of course, if you can get from *a* to *d* in *three* steps, but you can get from *a* to *c* in one, then you can get from *a* to *d* in two steps, so you can get from *a* to *d* in one step.) One way to build the transitive closure of the *Ethics'* graph, is to take these steps.

Step 1: Add all the edges from the adjacency table to the transitive closure.
Step 2: Count the edges in the transitive closure.
Step 3: Search the transitive closure for all cases where <a,b> and <b,c> are already in the transitive closure. If <a,c> is not already in the transitive closure, add it.
Step 4: After examining the entire transitive closure, count its edges.
Step 5: If the count obtained in Step 4 is greater than the count in Step 2, repeat Steps 2 and 3; otherwise, we are finished.

Again, just to make clearer how we use the transitive closure: it is very easy to see whether a graph has an edge <*m*,*n*>: you just look in its adjacency table for the ordered pair <*m*,*n*>. On the other hand, seeing whether the graph contains a path can be very laborious because you have to check a node's children, grandchildren, great-grandchildren, and so on until you have either found your target or reached only leaves. But every *path* in the original graph is an *edge* in the graph's transitive closure, so the task of looking for paths in the graph is reduced to the (much easier) task of looking for an edge in the transitive closure.

The transitive closure opens up a whole class of questions about how Spinoza's positions are related. There are questions about the coherence and completeness of the system: Does every pair of proved propositions have a common ancestor? Does every pair of leaves have a common ancestor? Does every pair of assumptions have a common descendent? What portion of the system would survive the removal of each assumption? There are questions relating to particular portions of the system: How are the ethical portions of the graph related to the metaphysical? These are all questions about paths in the original graph.

The transitive closure of the *Ethics* graph allows us to explore some interesting classes of nodes. For instance, the descendents of 1A04 are the 300 nodes that have 1A04 as parent

in the transitive closure. The axiomatic ancestors of 4P46 are the 37 statements that have in-degree 0 and 4P46 as child in the transitive closure.

Finally, there are a number of sub-graphs that warrant exploration. As a typical instance, suppose we begin with the nodes that descend from 1A04 (which we get from the transitive closure, as just explained). Next we build our new sub-graph's adjacency table in this way: from the *Ethics*' adjacency table we draw all the edges that have both parent and child among the descendents of 1A04. The result is a connected sub-graph of the *Ethics*.[3] From this sub-graph we can learn some curious things about the *Ethics*: for instance, Spinoza derives 5P22 from 1A04 and 1P03. But 1P03 is derived solely from 1A04. This should focus our attention on 1P03: what contribution does it make to the proof of 5P22? And if the contribution is indispensible, then what else, besides 1A04, really lies behind 1P03?

The tools

As I mentioned at the beginning, this is a scouting expedition. I hope to show where to look for fresh Spinozistic issues and to provide some tools necessary to explore them.

Formally, a graph is a structure consisting of a set of nodes and a binary relation over that set. A binary relation is a set of ordered pairs, $\langle a, b \rangle$, where *a* and *b* are both in the set. One natural way to represent a graph is as a pair of tables. The first is simply a list of the nodes. In the table I supply, the "Nodes" table contains one row for each statement in the ethics and columns for the statement's name (according to our naming convention), a paraphrase of the statement, and two columns calculated from the adjacency table, namely, the in-degree and out-degree of the node.

The second table is the "adjacency table." Each record contains a field called 'Parent' and a field called 'Child.' Each time Spinoza says, "This follows from that" we generate a record in the adjacency table with "that" as parent and "this" as child. The in-degree of a statement is the number of times it appears as child in the adjacency table; the out-degree is the number of times it appears as parent.

Many of the things we would like to explore in the graph are more easily examined in a *different* graph called the graph's "transitive closure." If there is a *path* in the graph from *a* to *z*, then there is an *edge* in the graph's transitive closure connecting *a* and *z*. We build the transitive closure table by first adding all the records in the adjacency table (so we have all the parent-child records). Then, we add the grandparent, grandchild records, the great-grandparent, great-grandchild records, and so on until there are no more records to add.

The transitive closure allows us to find all the statements that descend from 1A04: it is the class of child nodes in the transitive closure with 1A04 as parent. (Notice that there are 9 immediate consequences of 1A04, but 300 that inherit from it eventually.) Similarly, we can find all statements that contribute somehow to the proof of 5P34, namely the parent nodes in the transitive closure with 5P34 as child. (There are 69 of

them.). The axioms that contribute to 5P34 are the nodes in the transitive closure with in-degree = 0 and 5P34 as child. (There are 21 of them.). Finally, the proof that the original graph is acyclic is that its transitive closure contains no record of the type <*x*, *x*>. The transitive closure of *Ethic's* graph contains 22,540 edges, as contrasted with the 1,035 edges in the graph itself.

The present project began in 1981, when I attempted to build the adjacency table on poster-board with a ball-point pen. This proved utterly impractical: because the graph is directed and acyclic, I had to deal with only about half the 184,900 cells in the square matrix of statements, but that was still too much for my resources. In addition, it was very hard to trace the paths through the graph which were my chief interest. I could not resurrect the project until decent relational databases became available in the 1990's. The present project was built in Microsoft Access.

The present project was completed before I found Ron Bombardi's scanned version of the text. (http://frank.mtsu.edu/~rbombard/RB/Spinoza/ethica1.html). Both Bombardi and I have faced the same problem: I had to build the adjacency table by reading the text and entering the pairs of statements into the table by hand; Bombardi performed the same task to get the hyperlinks in his text. Either of us could have overlooked some citations; but since we worked independently, it is not likely that we each made the same mistakes. Hence, a comparison of our results will improve both. My friend and colleague, Mike Beane of Sebasticook Valley Hospital, has kindly built a Perl script to count the hyperlinks in each proposition, so that we may, at least, be sure that Bombardi and I saw the same number of references to earlier statements in the text. We were not able to verify that Bombardi and I have seen exactly the same references because Bormardi's hyperlinks do not use consistent names for earlier statements.

My three tables are attached as XML files. XML has become the de facto standard for data exchange of late years. While XML files are certainly verbose, they are convenient since they can be imported directly into most data management applications, such as Microsoft Excel and Access, and the various SQL database managers.

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[1] A graph can fail of connectivity without containing nodes with in-degree and out-degree = 0. It might, for instance, contain two internally connected regions—two islands, as it were. *Ethics* does not contain such islands. The proof of this is rather tedious and involves examining an undirected graph derived from the directed graph we are discussing.

[2] Spinoza adds two axioms after 2P13 and names them Axiom 1 and Axiom 2, creating a clash with the like-named axioms at the beginning of the part. I have renumbered them 2A07 and 2A08.

[3] This sub-graph has a single element with in-degree zero, which we might call the sub-graph's 'root' or 'unit'. Moreover, there is a path from each node to the root, and consequently every pair of nodes has an upper bound…features that the sub-graph shares with trees and with upper semi-lattices, respectively. Nevertheless sub-graphs of the *Ethics* graph are not ordinarily either trees or upper semi-lattices. They are not trees because there are usually multiple paths from a given node to the root. Upper semi-lattices can have multiple paths to the unit, but every pair of nodes must have a least-upper-bound: that is, there is a particular upper bound for the pair such that all other upper bounds are between it and the unit. In the case of the sub-graph of descendents of 1A04, 1P05 and 1P07 are parents of 1P08 and each is also a parent of 1P12, so 1P05 is an upper bound of the pair 1P08 and 1P12, and 1P07 is also an upper bound of 1P08 and

1P12; but neither 1P05 nor 1P07 is “above” the other. Moveover, no other upper bound of 1P08 and 1P12 can be “below” 1P05 or below 1P07 (since each is an immediate parent of 1P08 and 1P12), so the pair <1P08, 1P12> has upper bounds, but no least-upper-bound.