Topic 1-a
The Valuation of Cash Flows and Investment Returns

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Topic 1-a
The Valuation of Cash Flows and Investment Returns

areas to be covered

Time Value of Money
Valuing Cash Flows
Coupon, Single Payment, Fixed Payment and Discount Bonds
Expected versus Realized Yields
Realized Return over Multiple Periods
Returns over Successive One-Year Periods

Cash Flow Characteristics of Securities

In general, a security is a promise pay cash at periodic times in the future
- The cash flow characteristics differentiate various "types" of securities

There are four general cash flow characteristics
- Single payment instruments
- Coupon instruments
- Fixed payment instruments
- Discount instruments

Question:

How are these various cash flows valued?
How are they compared?

To answer this, we must begin with the time value of money.

The Time Value of Money

Basic premise: a dollar today is worth more than a dollar a year from now.

Why?
- Today's dollar may be invested to provide more dollars in the future.
- Also, you must wait (forgo spending) to get tomorrow's dollar.
- Together, these mean that people have a positive time preference. They will forgo spending today only if they can have more spending in the future.

Time Value of Money

If a dollar can be invested, how much will you have when the "contract matures?"

$$FV = PV \times (1 + I)^n$$

- PV = present value of an amount invested (or the value of money today)
- i = the interest rate
- n = number of compounding periods (number of periods in which interest is earned on interest)
- FV = final value (or the value of the investment when the contract matures in n periods)

Time Value of Money

Consider an investment of $100 for one year at a rate of 5%

$105 = 100 (1.05)^1$

Now if the investment is repeated a year from now

$110.25 = 105 (1.05)^1$

This is the same as

$110.25 = 100(1.05)^2$

The Time Value of Money

The process can be reverse.

- If a 100 is promised a year from now, what is that expected cash flow worth today?

To answer this, you need a interest rate. Suppose that the market rate is 5%

$PV = FV / (1 + i)^1$ or $100 / (1.05)$ or 95.24

Why? because you could take 95.24, invest it at 5% and receive 100 a year from now

Time Value of Money

Next question: what would the 100 be worth today, if it would not be received for 2 years, and the market rate is 5%

$PV = FV / (1.05)^2$ or $100 / (1.05)^2$ or 90.79

Time Value of Money

Any future cash flow has a value today.

The value today depends upon:

- The market rate, and
- The length of time until the cash flow is received.

Time Value of Money

Next, suppose that you are promised 100 next year and 100 in two years.

What are the two cash flows worth today?

- $PV1 = 100 / (1.05)^1$
- $PV2 = 100 / (.005)^2$

- $PV = PV1 + PV2$
- $PV \quad 100 / (1.05)^1 + 100 / (1.05)^2$

Why? present values are additive

Single Payment Instrument

Suppose that you invest 10000 in a five year CD with a 6.875% rate, compounded annually. How much will you have in five years?

$FV = PV \times (1.06875)^5$

$13,943.70 = 10000 \times 1.39437$

Next, suppose that market rates fall to 6.25% after you buy the CD. If your were to sell the CD, how much would it be worth?

$PV = FV / (1.0625)^5$ or ?

Coupon Bonds

Definition: A contractual obligation to make periodic interest payments for a fixed period; at maturity the lender is repaid the original amount that was borrowed.

Terms:

- *Face value:* original amount that was borrowed. This is also known as the principal.
- *Coupon rate:* a percentage of the face value.
- *Coupon payment:* periodic interest payment. It is the product of the coupon rate and the face value
- *Maturity:* date at which the contract matures
- *Term to maturity:* length of time until the contract matures

The Time Value of Money and the Value of Bonds

Since a financial instrument is a promise to make periodic cash payments, its value today depends upon

- The timing and size of those periodic payments, and
- The market rate of interest.

An Example

Suppose that you consider buying a bond with the following characteristics:

Face value:	10,000
Coupon rate:	6.875%
Maturity:	August 31, 2000
Today's date:	August 31, 1995

What are the cash flow characteristics of this bond:

Cash Flow Characteristics

Periodic interest payment

- .06875 x 10,000 = 687.5
- 687.5 is paid each year for five years

Face value

- 10,000
- Face value is received at the end of the fifth year

Combined the cash flow would be:

Yr1	Yr2	Y3	Yr4	Yr5
687.5	687.5	687.5	687.5	10687.5

What is this cash flow stream worth today?

Market Value of the Cash Flow

One additional piece of information is needed.

- The yield to maturity or the market rate of interest

Yield to maturity:

- The rate of discount that equates the present value of a cash flow stream to its market price.

Suppose that the *yield to maturity* is 6.875%. What is the market price?

- Price = ?

HINT: n=5 pmt = 687.5 FV=10000
i=6.875% (i=yield to maturity on the calculator)

Market Value of Cash Flow

Next suppose that the yield to maturity falls to 6.25%

What are the cash flow characteristics of this security?

n = 5

FV = 10000

pmt = ?

i = 0.0625

PV = ?

Market Value of Cash Flow

Next suppose that the yield to maturity rises to 7.25%

What are the cash flow characteristics of this security?

n = 5

FV = 10000

pmt = ?

i = 0.0725

PV = ?

Fixed Payment

Suppose

- 10000 is borrowed and is to be paid back over five years,
- The rate is 6.875%,
- Each cash flow will represent some principal and interest on the outstanding balance; the loan balance will be zeros at the end of five years,
- This type of instrument is an amortizing asset, and
- Car loans and mortgages are amortizing assets

What is the monthly cash flow to the lender or the buyer of this security?

Fixed Payment

Cash flow characteristics of this asset:

Yr1	Yr2	Yr3	Yr4	Yr5
2430.75	2430.75	2430.75	2430.75	2430.75

The outstanding balance at the end of year five = 0

If the YTM is 6.875,

- The market price of this instrument is:

HINT: n = 5 pmt = 2430.75 FV = 0
i = 6.875%

Fixed Payment

What would be the market price if the YTM falls to 6.25%

n = 5

pmt = 2430.75

FV = 0

i = 6.25%

PV = ?

If the YTM rises to 7.25%

Cash Flow Characteristics

Note the contrast between the cash flow characteristics of the single payment asset, the coupon asset and the fixed payment asset.

Discount Security

Suppose:

- There is a promise to make you a single payment of 10000 in 30 years

Cash flow

Yr1	Yr2	Yr3	Yr4	Yr5	....	Yr28	Yr29	Yr30
0	0	0	0	0		0	0	10000

This instrument has the same cash flow characteristics as a single payment instrument.

Discount Security

BUT, unlike the single payment instrument:

- The final value of this instrument is specified
  - In the single payment instrument, the final value depended upon the instrument's stated interest rate
- The initial investment depends upon the instrument's YTM
  - In the single payment instrument, the initial investment is specified

Discount Security

How much would you be willing to "pay" for the promise of 10000 in 30 years?

That is, what is the market price of this cash flow?

You need the YTM.

Suppose the YTM is 6.875%

The Price of a Discount Security

HINT:

n	=	30
pmt	=	0
FV	=	10000
i	=	6.875
pv	=	?

Suppose the YTM falls to 6.25%

Market price = ?

Suppose the YTM rises to 7.25%

Market price = ?

Common Thread Among Securities

The yield to maturity is a basis of comparing securities with different cash flows.

- This is an industry standard

But the YTM has several important assumption.

- The investor is assumed to hold the security until maturity,
- The investors is assumed to reinvest all cash flows from the instrument
- The investment of all cash flows is made at the YTM rate

YTM

Thus, the YTM is an expected return.

It is the return that would be achieved if

- The security is held to maturity
- All cash flows are reinvested
- And that these cash flows are invested at the YTM rate

If any other conditions hold, the realized rate will differ from the YTM

YTM Versus Expected Return

YTM is an expected return on the assumption that:

- The security is held until maturity
- Coupons are reinvested at the YTM

However, suppose that the investor has a shorter holding or investment period than the maturity of the asset

In this case, an expectation on the likely price of the security at the end of the investment period

Expected Return

Consider the following situation. An investor buys a five-year bond with the following characteristics:

FV	10000
Coupon rate	7.375%
Maturity:	Exactly five years
YTM	7.375%

The investor plans to sell the security in two years

The investor thinks that the price will be 101:02 in two years

Expected Return

What is the expected return over the 2-year investment horizon?

PV	$10,000
Coupon payment per period	$737.50
Presumed price	$10,106.25
Number of compounding periods	2
Expected yield	*7.8861%*

Problem: expected yield assumes that the coupon payments will be reinvested at the expected yield

Expected Return
Dollar-Weighted Rate of Return

The previous calculation is a dollar-weighted rate of return.

It is akin to the YTM:
- Compound annual rate of return expected over the investment horizon
- Rate that equates the ***expected*** cash flow to the original market price of the cash flow

Also know as internal rate of return

Expected Return
Dollar-Weighted Rate of Return

Another example --suppose you bought the following security

Market price	98:25
Coupon rate	6.25%
Face value	$100,000
Maturity	Exactly 5 years
Interest paid annually	

The YTM at the time of purchase is ?

Coupon payment	$3,125 every six months
Purchase price	$98,781.25
YTM	6.5397%

Expected Return
Dollar-Weighted Rate of Return

Next, the security is sold after two years for 101:02

Cash inflow over the two year period:

2 interest payments of	$6250
Terminal cash inflow	$101,062.50
Cash outflow:	$98,781.25

Realized return over the two-year investment period

7.44%

This assume reinvestment of the interest at the IRR rate

Realized Return

Realized return depends on financial condition that develop over the investment horizon

This will affect
- The sale price of the security at the end of the 2-year investment horizon
- The rate at which coupons (interim cash flows) will be reinvested

Realized Return
An Illustration

Consider the following situation. An investor buys a five year bond with the following characteristics:

FV	10000
Coupon rate	7.375%
Maturity:	Exactly five years
YTM	7.375%

The investor sells it one year later, but the YTM is 7.625%.

What is the realized return over the one-year investment period?

Realized Return
An Illustration

Simple dollar return:

Cash flow during investment period
+ Sales Price
+ the coupon payment
- Purchase Price

Cash flow during investment period = ?
Sale price = ?
Coupon payment = ?
Purchase price = ?

Realized Return
An Illustration

Simple rate of return

Realized cash inflow / Purchase price

? / ?

= ?

Realized Return
An Illustration

Cash inflow

Actual sale price	9,916.50
Coupon payment	737.50

Cash outflow

Purchase price	10,000

Simple realized return:
[(9916.50 + 737.50) / 10000] x 100 = 1.0654
Realized return = 6.54%

Multi-Period Realized Returns

Realized return over a multi-period depends upon the rate at which the coupon is reinvested

Total Return Over Successive One-Year Periods

Returns must be evaluated on an ongoing basis

Investment return can be calculated for any period, even if the security is not sold

Consider the following bond

Coupon rate	6.25%
Face value	100,000
Maturity	Exactly 5 years
Purchase price	98:25
Interest paid annually	

Total Return Over Successive One-Year Periods
Continued

YTM at time of purchase 6.5436%

At the end of the first year, the security is worth 97:25, what is the "return" for the first year?

End of year value $97,781.25

Interest $6250

One year return:

((6250 + 97781.25)/$98,781.25)*100

5.314774%

Total Return Over Successive One-Year Periods
Continued

At the end of the second year, the price is 101:02

What is the one-year return

Value at the end of year two $101,062.50

Interest $6250

Value at the end of year one $97,781.25

One year return on the market value of the security at the start of year two: 9.747523%

Total Return Over Seccessive One-Year Periods
Continued

The previous analysis is inadaquate. *WHY?*

It did not consider the effect of the interest received at the end of the first period.

- This interest is available for investment and ssue:
- The reinvestment rate will influence the total return experienced during the second year

Total Return Over Seccessive One-Year Periods
Continued

At the end of year one $6250 in interest was received.

The interest must be invested

Suppose that the interest is invested for ***one year*** at the market rate of 6% at the end of year 1

At the end of year two, interest on the interest would be

6250 x .06 = 375

Total Return Over Seccessive One-Year Periods
Continued

Cash inflow at the end of year two attributable to the original security:

Market value	101062.50
Current interest	6250
Compounded interest from previous period	6625.00
TOTAL cash inflow at end of year two	113937.50
Total at start of year 2	97781.25
	6250
	104031.25

Total Return Over Seccessive One-Year Periods
Continued

Total return over second year

[113937.50 / 104031.25] = 1.0952

or

9.52%

Compound *total* annual return over the two-year period

$[1.0531 \times 1.0952]^{1/2}$ = 7.39%