Hypothesis Testing & One Sample *Z* Test

Type I Error = rejecting the null hypothesis when it is true

Type II Error = retaining the null hypothesis when it is false

Power = the probability of rejecting the null hypothesis when it is false

Ways of increasing power

Increase alpha – increases power slightly, but also increases risk of Type I Error

Use a directional hypothesis – increases power slightly if predict the correct direction; power approaches zero if predict the wrong direction.

Increase the sample size – Recommended approach

Compare extremes of the population – results do not generalize to the moderates in the population.

Control extraneous variables – can increase power, but changes hypothesis being tested

Scenario 1. SAT scores are known to be normally distributed with a national mean of 1000 and a standard deviation of 100. The SAT scores of 144 incoming freshmen were obtained; the average SAT score was 1075. Calculate and interpret the 95% confidence interval for the mean of incoming freshmen.

Standard Error = $\sqrt{\frac{10000}{144}}$ = 8.333

Critical Values: -1.96, 1.96

1075 + 1.96(8.33) = 1058.67
1075 - 1.96(8.33) = 1091.32

We are 99% confident that the interval (1058, 1091) covers the average SAT for the freshman population (M = 1075, n = 144).

Scenario 2. Female weight is known to be normally distributed with a national average of 150 and a standard deviation of 20 pounds. A physician at a private university wants to estimate the 99% confidence interval for the average weight of females attending the university. Twenty-five females were weighted and found to have an average of 120 pounds. Calculate and interpret the 99% confidence interval for the weight of university women.

Standard Error = $\sqrt{\frac{400}{25}} = 4.00$

Critical Values: -2.58, 2.58

120 + 2.58(4.0) = 130.32
120 – 2.58(4.0) = 109.68

We are 99% confident that the interval (109.68, 130.32) covers the average weight of the population of females at the university (M = 120.00, n = 25).

Scenario 3. A child psychologist wanted to know whether children lie more frequently than adults. Previous research with adults indicate they tell an average of five lies per day with a standard deviation of two. Thirty children were observed by the psychologist for one day, and the number of lies the children told was measured. The children told an average of 3.5 lies per day. Use an alpha of .01 to test the null hypothesis.

Null: Children do not lie more frequently than adults (=, <).
Alternative: Children lie more frequently than adults (>).

Directional hypotheses, Critical Value = 2.33.

Standard Error = $\sqrt{\frac{4}{30}} = 0.3651$

$Z = \frac{3.5 - 5}{0.3651} = -4.11$

Since $Z = -4.11$ does not exceed the critical value of 2.33, the null hypothesis is retained.

The one sample *Z* test with an alpha of .05 indicated children ($M = 3.50$, $n = 30$) do not lie more frequently than adults (*Population Mean* = 5.00, *Population SD* = 2.00), $Z = -4.11$, $p > .01$.

Scenario 4. A researcher wanted to estimate the actual IQs of persons receiving academic scholarships. Sixty-four persons receiving academic scholarships were randomly selected and given an IQ test. The sample average was 115. Within the United States, IQ is known to have an average of 100 with a standard deviation of 15. Do persons on academic scholarships have significantly different IQs than persons in the general population? Use an alpha of .05 to test the null hypothesis.

Null: Scholarship students have similar IQs to the general population (=).
Alternative: Scholarship students have different IQs than the general population (≠).

Nondirectional hypotheses, Critical values: -1.96, 1.96

$$\text{Standard Error} = \sqrt{\frac{225}{64}} = 1.875$$

$$Z = \frac{115 - 100}{1.875} = 8.00$$

Since $Z = 8.00$ exceeds the critical values of -1.96, 1.96, the null hypothesis is rejected.

The one sample *Z* test with an alpha of .05 indicated scholarship students ($M = 115.00$, $n = 64$) have different IQs than the general population (*Population Mean* = 100, *Population SD* = 15), $Z = 8.00$, $p < .05$.

Scenario 5. A researcher believes that pet owners have lower stress in their lives than non-pet owners. To investigate her idea, the researcher randomly sampled 50 pet owners and administered the Stress Inventory to them. The 50 pet owners had an average of 80 on the Stress Inventory. Previous research has shown that non-pet owners have a mean of 90 and a standard deviation of 30 on the Stress Inventory. Use an alpha of .01 to test the null hypothesis.

Null: Pet owners do not have less stress in their lives than non-pet owners (=, >).
Alternative: Pet owners have less stress in their lives than non-pet owners (<).

Directional Hypotheses, Critical Value: -2.33.

Standard Error = $\sqrt{\frac{900}{50}}$ = 4.2426

$Z = \frac{80 - 90}{4.2426}$ = -2.36

Since the Z = -2.36 exceeds the critical value of -2.33, the null hypothesis is rejected.

The one sample Z test with an alpha of .05 indicated pet owners (M = 80.00, n = 50) have less stress than non-pet owners (*Population Mean* = 90, *Population SD* = 30), Z = -2.36, $p < .01$.