# One-Way Analysis of Covariance
# (One-Way ANCOVA)

**The ideal method of adjusting for possible extraneous variables is to randomly assign participants to the treatment conditions.**

- Eliminates all pre-existing group differences
- Not always possible, ethical, or feasible

**The researcher may statistically control for one or more extraneous variables by including them as covariates in the analysis.**

- Does not eliminate all pre-existing group differences, just those included as covariates
- Matching groups on one dimension may mean you are creating differences along a second dimension.

## One-Way ANOVA Research Questions

- ❑ Is gender (female, male) a significant predictor of salary?
- ❑ Can we predict students' reading scores using type of school system (public, private)?

## One-Way ANCOVA Research Questions

- ❑ Is gender (female, male) a significant predictor of salary when controlling for years of experience?
- ❑ Can we predict students' reading scores using type of school system (public, private) after adjusting for total household income of the parent(s) and the average age of the parents?

## Examples of ANCOVA Designs

<table>
<tr>
<td>One-Way ANCOVA<ul><li>One independent variable (between-subjects)</li><li>One dependent variable</li><li>One or more quantitative covariates</li></ul>Two-Way ANCOVA<ul><li>Two independent variables(between-subjects)</li><li>One dependent variable</li><li>One or more quantitative covariates</li></ul></td>
<td>One-Way Repeated Measures ANCOVA<ul><li>One independent variable (within-subjects)</li><li>One dependent variable</li><li>One or more quantitative covariates</li></ul>Two-Way Repeated Measures ANCOVA<ul><li>Two independent variables (1 or both may be within-subjects)</li><li>One dependent variable</li><li>One or more quantitative covariates</li></ul></td>
</tr>
</table>

| Type of Practice | X = Initial Success | Y = Success after Practice |
|---|---|---|
| Unstructured | 4 | 5 |
| Unstructured | 2 | 4 |
| Unstructured | 3 | 9 |
| Structured | 3 | 11 |
| Structured | 4 | 10 |
| Structured | 8 | 15 |

The **Reduced Model** is $Y_{ij} = \mu + \beta X_{ij} + e_{ij}$

$$slope = \frac{\sum(\mathrm{X} - \overline{\mathrm{X}})(\mathrm{Y} - \overline{\mathrm{Y}})}{\sum(\mathrm{X} - \overline{\mathrm{X}})^2} = \frac{SP}{SS_X}$$ based on the grand means.

The <u>slope</u> and the <u>intercept</u> are calculated by **<u>ignoring group membership</u>**.

**Slope Calculation for the REDUCED MODEL**

| $Y_{ij}$ | $X_{ij}$ | $(Y - \overline{Y})$ | $(X - \overline{X})$ | $(X - \overline{X})^2$ | $(Y - \overline{Y}) * (X - \overline{X})$ |
|---|---|---|---|---|---|
| 5 | 4 | | | | |
| 4 | 2 | | | | |
| 9 | 3 | | | | |
| 11 | 3 | | | | |
| 10 | 4 | | | | |
| 15 | 8 | | | | |
| Grand Mean<br>9 | Grand Mean<br>4 | 0<br>Always | 0<br>Always | $SS_X$ | SP |

$$slope = \frac{SP}{SS_X} =$$

$$\hat{\mu} = \text{int}ercept = \overline{Y} - Slope\left(\overline{X}\right) =$$

The **Reduced Model** is:

**Predicted Scores & Errors for the REDUCED MODEL**

| $Y_{ij}$ | $X_{ij}$ | Predicted Y | Errors $(Y - \hat{Y})$ | Squared Errors $(Y - \hat{Y})^2$ |
|---|---|---|---|---|
| 5 | 4 | | | |
| 4 | 2 | | | |
| 9 | 3 | | | |
| 11 | 3 | | | |
| 10 | 4 | | | |
| 15 | 8 | | | |
| Grand Mean<br>9 | Grand Mean<br>4 | | | $E_R$ |

The **Full Model** is $Y_{ij} = \mu + \alpha_j + \beta X_{ij} + e_{ij}$

The **Full Model** has a single slope parameter, but allows for an effect of the treatment variable.

$$slope = \frac{\sum_{j=1}^{a}\sum_{i=1}^{n}(X_{ij} - \overline{X}_j)(Y_{ij} - \overline{Y}_j)}{\sum_{j=1}^{a}\sum_{i=1}^{n}(X_{ij} - \overline{X}_j)^2} = \frac{SP}{SS_X}$$ is based on the group means.

Thus, the **Full Model** in the ANCOVA allows the intercepts of the regression lines to vary with each group, but the slope parameter is the same for each group.

**Slope Calculation for the FULL MODEL**

| $Y_{ij}$ | $X_{ij}$ | $(Y - \overline{Y}_j)$ | $(X - \overline{X}_j)$ | $(X - \overline{X}_j)^2$ | $(Y - \overline{Y}_j) * (X - \overline{X}_j)$ |
|---|---|---|---|---|---|
| 5 | 4 | | | | |
| 4 | 2 | | | | |
| 9 | 3 | | | | |
| *Mean = 6* | *Mean = 3* | | | | |
| 11 | 3 | | | | |
| 10 | 4 | | | | |
| 15 | 8 | | | | |
| *Mean = 12* | *Mean = 5* | | | | |
| | | 0<br>Always | 0<br>Always | $SS_X$ | SP |

$$slope = \frac{SP}{SS_X} =$$

$\hat{\mu} + \hat{\alpha}_1$ = intercept for unstructured practice = $\overline{Y}_1 - Slope(\overline{X}_1) =$

$\hat{\mu} + \hat{\alpha}_2$ = intercept for structured practice = $\overline{Y}_2 - Slope(\overline{X}_2) =$

The Full Model for unstructured practice is:

The Full Model for structured practice is:

**Predicted Scores & Errors for the FULL MODEL**

| $Y_{ij}$ | $X_{ij}$ | Predicted Y | Errors $(Y - \hat{Y})$ | Squared Errors $(Y - \hat{Y})^2$ |
|---|---|---|---|---|
| 5 | 4 | | | |
| 4 | 2 | | | |
| 9 | 3 | | | |
| *Mean = 6* | *Mean = 3* | | | |
| 11 | 3 | | | |
| 10 | 4 | | | |
| 15 | 8 | | | |
| *Mean = 12* | *Mean = 5* | | | |
| | | | 0 Always (w/in rounding) | $E_F$ |

$df_R = N - 2$ and $df_F = N - a - 1$

$df_{numerator} = a - 1$ and $df_{denominator} = df_F = N - a - 1$ $\quad Fcriticalvalue =$

$$F = \frac{(E_R - E_F)/(df_R - df_F)}{E_F / df_F}$$

Interpretation

$$Partial\ \eta^2 \text{for } A \text{ factor} = \frac{SS_A}{SS_A + SS_{Error}}$$

$$\text{Partial } \hat{\omega}^2 = \frac{(a-1)(F_A - 1)}{(a-1)(F_A - 1) + N}$$

**<u>Calculation of Adjusted Means (</u>**Use the Slope from the Full Model)

$\overline{Y}_j' = \overline{Y}_j - b_w(\overline{X}_j - \overline{X})$

Adjusted Unstructured Mean =

Adjusted Structured Mean =

## Pairwise Comparisons with ANCOVA

*F* Statistic for Pairwise Contrasts on the Adjusted Means

$$F = \frac{\hat{\psi}^2}{\text{var}(\hat{\psi})} \quad \text{where} \quad \begin{aligned} \hat{\psi} &= \bar{Y}_l' - \bar{Y}_m' \\ &= \bar{Y}_l - \bar{Y}_m - b_w(\bar{X}_l - \bar{X}_m) \end{aligned} \quad \text{and} \quad \bar{Y}_j' = \bar{Y}_j - b_w(\bar{X}_j - \bar{X})$$

$$\text{and} \qquad \textbf{var}(\hat{\psi}) = \frac{E_F}{df_F}\left[\frac{1}{n_l} + \frac{1}{n_m} + \frac{(\bar{X}_l - \bar{X}_m)^2}{\sum_{j=1}^{a}\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2}\right]$$

**Bonferroni and Sidak Methods for the Adjusted Means Are Available in SPSS.**

⇒ Analyze ⇒ General Linear Model ⇒ Univariate ⇒ Options ⇒ Compare Main Effects
⇒ Confidence Interval Adjustment ⇒ Sidak

**Hand calculations will be required if you want to use the Tukey or Scheffe methods.**

1. Have SPSS generate the adjusted means and standard errors.

   ⇒ Analyze ⇒ General Linear Model ⇒ Univariate ⇒ Options ⇒ Compare Main Effects
   ⇒ Confidence Interval Adjustment ⇒ LSD (none)

2. Calculate $F = \left(\frac{\text{Mean Difference}}{\text{Std. Error}}\right)^2$ for each pairwise comparison.

3. Find the appropriate Tukey or Scheffe *F* critical value. Use denominator *df = N – a - # covariates.*

**ONE-WAY ANCOVA: Is type of school system (public, private, university) a significant predictor of children's math scores after adjusting for the highest education level of the parent(s)?**

SPSS Instructions
→ Analyze →General Linear Model→Univariate
Dependent Variable: Math
Fixed Factors: School
Covariate: Educ
√ Estimates of Effect Size
√ Descriptive Statistics
→Options
Display Means For: School
√ Compare Main Effects
Confidence Interval Adjustment: Sidak

**Univariate Analysis of Variance**

**Between-Subjects Factors**

| | | Value Label | N |
|---|---|---|---|
| school | 1.00 | public | 15 |
| | 2.00 | private religious | 15 |
| | 3.00 | university school | 15 |

**Descriptive Statistics**

Dependent Variable:math score

| school | Mean | Std. Deviation | N |
|---|---|---|---|
| public | 50.8667 | 12.05859 | 15 |
| private religious | 63.1333 | 11.64883 | 15 |
| university school | 71.8000 | 14.43804 | 15 |
| Total | 61.9333 | 15.20526 | 45 |

**Tests of Between-Subjects Effects**

Dependent Variable:math score

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
|---|---|---|---|---|---|---|
| Corrected Model | 3486.333[a] | 3 | 1162.111 | 7.126 | .001 | .343 |
| Intercept | 5965.228 | 1 | 5965.228 | 36.578 | .000 | .471 |
| educ | 167.400 | 1 | 167.400 | 1.026 | .317 | .024 |
| school | 1767.881 | 2 | 883.941 | 5.420 | .008 | .209 |
| Error | 6686.467 | 41 | 163.085 | | | |
| Total | 182781.000 | 45 | | | | |
| Corrected Total | 10172.800 | 44 | | | | |

a. R Squared = .343 (Adjusted R Squared = .295)

**Estimated Marginal Means**

**school**

**Estimates**

Dependent Variable:math score

| school | Mean | Std. Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| public | 52.419[a] | 3.636 | 45.076 | 59.761 |
| private religious | 62.959[a] | 3.302 | 56.290 | 69.627 |
| university school | 70.423[a] | 3.567 | 63.220 | 77.626 |

a. Covariates appearing in the model are evaluated at the following values: years educ. of parent = 18.8222.

**Pairwise Comparisons**

Dependent Variable:math score

| (I) school | (J) school | Mean Difference (I-J) | Std. Error | Sig.[a] | 95% Confidence Interval for Difference[a] | |
|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound |
| public | private religious | -10.540 | 4.965 | .115 | -22.898 | 1.819 |
| | university school | -18.004* | 5.487 | .006 | -31.662 | -4.346 |
| private religious | public | 10.540 | 4.965 | .115 | -1.819 | 22.898 |
| | university school | -7.464 | 4.812 | .338 | -19.442 | 4.513 |
| university school | public | 18.004* | 5.487 | .006 | 4.346 | 31.662 |
| | private religious | 7.464 | 4.812 | .338 | -4.513 | 19.442 |

Based on estimated marginal means

a. Adjustment for multiple comparisons: Sidak.

*. The mean difference is significant at the .05 level.

**Univariate Tests**

Dependent Variable:math score

| | Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
|---|---|---|---|---|---|---|
| Contrast | 1767.881 | 2 | 883.941 | 5.420 | .008 | .209 |
| Error | 6686.467 | 41 | 163.085 | | | |

The F tests the effect of school. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

**Statistical Effect of Including a Covariate in the Model**

- Randomized Study – substantially reduces the unaccounted-for variance – leading to a more powerful test of group differences

- Nonrandomized Study – reduces the unaccounted-for variance AND answers the question 'Would the groups differ on the dependent variable if they had been equivalent on the covariate?'

**Assumptions for the ANCOVA Model**: $Y_{ij} = \mu + \alpha_j + \beta X_{ij} + e_{ij}$

1. The error scores $e_{ij}$ are independently distributed.
2. The error scores $e_{ij}$ are normally distributed.
3. The error scores $e_{ij}$ have an expected value of zero and a constant variance.

**Implications of Assumptions**

1. The conditional distribution of *Y* must be normally distributed if $e_{ij}$ is normally distributed.

2. The relationship between *Y* and *X* is presumed to be linear. If there is reason to suspect a polynomial relationship between *Y* and *X* then you can use regression to determine the appropriate way to model the covariate. **See page 9 for instructions.**

3. The separate within-group regression lines have the same slope because β is pooled across the groups.

   This assumption may be explicitly tested. **See page 10 for instructions.** You hope for a nonsignificant test…which would indicate the slopes are homogenous.

   If the test is significant, one of the following strategies may be used.

   - Conduct a one-way ANCOVA that allows heterogeneous slopes. Currently, the GLM procedure does not support it. Hand calculations would be required, but SPSS could be used to obtain most of the necessary information. See Maxwell and Delaney for more information.

   - One could also transform the covariate to a factor that has a few discrete levels and conduct a two-way ANOVA (see blocking information at the end of the notes) instead of a one-way ANCOVA. For example, school (public, private, university) X parent education (HS, BS, MS, PHD) would be the factors and math scores would be the dependent variable.

4. The covariate values are assumed to be fixed, not random. That is, statistical inferences can be made only to the *Y* values expected **at the particular *X* values included in the study**.

Nonsignificant ANOVA

Significant ANCOVA, quadratic covariate

Significant ANOVA

Significant ANCOVA, homogeneous (equal) slopes

Nonsignificant ANCOVA

Significant ANCOVA, heterogeneous (unequal) slopes

# TEST FOR POLYNOMIAL RELATIONSHIPS

SPSS Instructions
→Analyze→Descriptive Statistics→Descriptives
Variables: EDUC

**Descriptive Statistics**

| | N | Minimum | Maximum | Mean | Std. Deviation |
|---|---|---|---|---|---|
| years educ. of parent | 45 | 6.00 | 29.00 | 18.8222 | 4.96940 |
| Valid N (listwise) | 45 | | | | |

SPSS Instructions
→ Transform → Compute Variable
Target Variable: QUADEDUC
Numeric Expression: (educ-18.82)*(educ-18.82)
→ Analyze → Regression → Linear
Dependent: MATH
Independents: EDUC, QUADEDUC

## Regression

**Variables Entered/Removed**

| Model | Variables Entered | Variables Removed | Method |
|---|---|---|---|
| 1 | quadeduc, years educ. of parent[a] | . | Enter |

a. All requested variables entered.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .416[a] | .173 | .133 | 14.15443 |

a. Predictors: (Constant), quadeduc, years educ. of parent

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 1758.195 | 2 | 879.097 | 4.388 | .019[a] |
| | Residual | 8414.605 | 42 | 200.348 | | |
| | Total | 10172.800 | 44 | | | |

a. Predictors: (Constant), quadeduc, years educ. of parent

b. Dependent Variable: math score

**Coefficients[a]**

| Model | | Unstandardized Coefficients | | Standardized Coefficients | | |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | t | Sig. |
| 1 | (Constant) | 39.607 | 8.882 | | 4.459 | .000 |
| | years educ. of parent | 1.223 | .436 | .400 | 2.804 | .008 |
| | quadeduc | -.029 | .065 | -.064 | -.445 | .658 |

a. Dependent Variable: math score

**TEST FOR HOMOGENOUS SLOPES**

SPSS Instructions
→ Analyze →General Linear Model→Univariate
Dependent Variable: Math
Fixed Factors: School
Covariate: Educ
→Model
√ Custom
Model: School, Educ, School*Educ

## Univariate Analysis of Variance

**Between-Subjects Factors**

| | | Value Label | N |
|---|---|---|---|
| school | 1.00 | public | 15 |
| | 2.00 | private religious | 15 |
| | 3.00 | university school | 15 |

**Tests of Between-Subjects Effects**

Dependent Variable:math score

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Corrected Model | 3674.441[a] | 5 | 734.888 | 4.410 | .003 |
| Intercept | 5155.932 | 1 | 5155.932 | 30.943 | .000 |
| school | 207.778 | 2 | 103.889 | .623 | .541 |
| educ | 41.502 | 1 | 41.502 | .249 | .621 |
| School * educ | 188.108 | 2 | 94.054 | .564 | .573 |
| Error | 6498.359 | 39 | 166.625 | | |
| Total | 182781.000 | 45 | | | |
| Corrected Total | 10172.800 | 44 | | | |

a. R Squared = .361 (Adjusted R Squared = .279)

**Group Differences in the Covariate: Problems in Interpretation**

If the groups differ on the covariate, then interpretation of the results may be ambiguous. The ideal situation is for the groups to be similar on the covariate (i.e., nonsignificant test). **The instructions for testing are shown on page 12.**

Situations where Dependence occurs between the Treatment and Covariate

*Case I: Fluke Random Assignment*. Subjects were randomly assigned to groups and the covariate differs across groups because of chance variability. Thus, the rejection of the hypothesis is a type I error.

- This situation may be avoided by performing stratified random sampling.
- Regardless, it is acceptable to interpret ANCOVA when the dependence is caused by chance.

*Case II: Biased Assignment Procedure*. Subjects were assigned to treatment groups on the basis of their covariate scores. (e.g., if IQ < 100 headstart, else traditional schooling).

- The assumptions of ANCOVA **must** be met if the subjects were assigned to groups on the basis of their covariate scores. Heterogeneous slopes are likely.

*Case III: Treatment Affects the Covariate*. If the subjects' scores on the covariate are obtained after the onset of the treatment, then the covariate scores may be influenced by the treatment effect.

- It is very difficult to interpret the ANCOVA results because adjusting for the covariate also adjusts for the treatment effect!
- Generally, we do not interpret the ANCOVA in this situation.

*Case IV: Non-Random Assignment of Subjects to Groups*. If the design is really a quasi-experiment and not a true experiment, then the groups are pre-existing.

- We can only interpret the results as being *correlational* and not *causational*.
- Using ANCOVA to adjust for differences in the pre-existing groups may also artificially induce differences on other variables.
- Adjusting for one covariate does not eliminate any other covariates that might influence the pre-existing groups.

## TEST FOR GROUP DIFFERENCES ON THE COVARIATE

**SPSS Instructions**
→Analyze→Compare Means→One-Way ANOVA
Dependent List: EDUC
Factor: SCHOOL
→Options
√ Descriptives
→Post Hoc
√ Tukey …or another procedure

# Oneway

**Descriptives**

years educ. of parent

| | N | Mean | Std. Deviation | Std. Error | 95% Confidence Interval for Mean | | Minimum | Maximum |
|---|---|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound | | |
| public | 15 | 15.4667 | 4.18956 | 1.08174 | 13.1466 | 17.7868 | 6.00 | 21.00 |
| private religious | 15 | 19.2000 | 5.38782 | 1.39113 | 16.2163 | 22.1837 | 9.00 | 29.00 |
| university school | 15 | 21.8000 | 3.05193 | .78801 | 20.1099 | 23.4901 | 17.00 | 26.00 |
| Total | 45 | 18.8222 | 4.96940 | .74079 | 17.3292 | 20.3152 | 6.00 | 29.00 |

**ANOVA**

years educ. of parent

| | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Between Groups | 304.044 | 2 | 152.022 | 8.159 | .001 |
| Within Groups | 782.533 | 42 | 18.632 | | |
| Total | 1086.578 | 44 | | | |

# Post Hoc Tests

**Multiple Comparisons**

years educ. of parent
Tukey HSD

| (I) school | (J) school | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound |
| public | private religious | -3.73333 | 1.57614 | .057 | -7.5626 | .0959 |
| | university school | -6.33333* | 1.57614 | .001 | -10.1626 | -2.5041 |
| private religious | public | 3.73333 | 1.57614 | .057 | -.0959 | 7.5626 |
| | university school | -2.60000 | 1.57614 | .237 | -6.4292 | 1.2292 |
| university school | public | 6.33333* | 1.57614 | .001 | 2.5041 | 10.1626 |
| | private religious | 2.60000 | 1.57614 | .237 | -1.2292 | 6.4292 |

*. The mean difference is significant at the 0.05 level.

**Other Statistical Methods of Adjusting for Covariates**

Analysis of Residuals

- Regress the dependen variable on the covariate to obtain predicted values and prediction errors (residuals).
- Conduct a one-way ANOVA where the new dependent variable is the prediction errors from the regression model.

- This results in a test statistic that is not properly distributed. It is not recommended. See Maxwell and Delaney (2004) for more details.

Analysis of Gain Scores

- If the dependent variable and the covariate are measured on the same scale (e.g., pretest score = covariate, posttest score = dependent variable), then a gain score = posttest – pretest may be calculated.
- Conduct a one-way ANOVA where the gain scores are the dependent variable in the analysis.

- Randomized Design – the ANCOVA approach is almost always preferred because it is a more powerful test. The one advantage that gain scores have over ANCOVA is the slope parameter does not have to be estimated, so the gain score approach has 1 more degree of freedom.

- Nonrandomized Design – the ANCOVA and ANOVA with gain scores do not test the same hypothesis. When the ANCOVA question is not of interest, then the analysis of gain scores may be more appropriate.

ANOVA with Blocking

1. The continuous covariate is transformed into an additional factor with discrete levels (e.g., dichotomized).
2. Then, a two-way ANOVA would be conducted. One factor represents the IV for the research study; the other factor represents the covariate. The primary research question would still be the test of the independent variable.
3. The ANCOVA is generally preferred. ANCOVA uses fewer degrees of freedom and information about the covariate is not lost through converting it to discrete levels.