One-Way Repeated Measures ANOVA: Multivariate Approach

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Difference Variables

We create difference variables for the multivariate approach. If we have *a* levels of the repeated factor, then we need *a – 1* difference variables to represent the data.

The hypothesis of no differences between the treatments is a test that *all difference variables are simultaneously zero*.

The particular difference variables we choose do not matter for testing the null hypothesis as long as the comparisons are linearly independent.

The difference variables are probably correlated and the multivariate approach adjusts for this correlation among the difference variables.

Degrees of Freedom

With two or more factors…

- the denominator degrees of freedom for a multivariate *F* test may exceed the total sample size;
- the denominator degrees of freedom for a multivariate *F* test may be fractional (i.e., not a whole number).

F Values

There are four commonly reported methods of obtaining an *F* test when using the multivariate approach.

- Wilk's Lambda (acceptable—most commonly used)
- Pillai-Bartlett Trace (acceptable—most robust)
- Hotelling-Lawley Trace (unacceptable)
- Roy's Greatest Root (unacceptable)

One-way repeated measures ANOVA
The four approaches yield identical *F* values.

Two-way repeated measures ANOVA

- If $a = 2$ or $b = 2$, the four approaches yield identical *F* values.
- If $a > 2$ and $b > 2$, then the four approaches usually yield different *F* values.

When *a* has 2 levels…

Null Hypothesis:

Reduced Model: $D_i = 0 + e_i$

Alternative Hypothesis:

Full Model: $D_i = \mu + e_i$

	Time 1	**Time 2**
S1	8	10
S2	3	6
S3	12	13
S4	5	9
S5	7	8
S6	13	14

		Reduced Model	
	D = Time 2 - Time 1	**Predicted Value**	**Squared Prediction Errors**
S1	2		$(2-0)^2 = 4$
S2	3		$(3-0)^2 = 9$
S3	1		$(1-0)^2 = 1$
S4	4		$(4-0)^2 = 16$
S5	1		$(1-0)^2 = 1$
S6	1		$(1-0)^2 = 1$
	Mean = 2		**E_R = 32**

		Full Model	
	D = Time 2 - Time 1	**Predicted Value**	**Squared Prediction Errors**
S1	2		$(2-2)^2 = 0$
S2	3		$(3-2)^2 = 1$
S3	1		$(1-2)^2 = 1$
S4	4		$(4-2)^2 = 4$
S5	1		$(1-2)^2 = 1$
S6	1		$(1-2)^2 = 1$
	Mean = 2		**E_F= 8**

$$F = \frac{(E_R - E_F)/(df_R - df_F)}{E_F / df_F} \qquad df_R = n \qquad df_F = n - 1$$

$df_{numerator} = 1$ $df_{denominator} = n - 1$

$F_{Crit} =$

Interpretation:

SPSS Instructions

→ Analyze →General Linear Model →Repeated Measures
- Within-Subject Factor Name: TIME
- Number of Levels: 2
- → Add → Define
- Within-Subjects Variables(Time):
  - Time1(1)
  - Time2(2)
- →Options
  - Display Means for: TIME
  - √ Estimates of Effect Size
  - √ Descriptive Statistics

General Linear Model

Within-Subjects Factors

Measure:MEASURE_1

Time	Dependent Variable
1	Time1
2	Time2

Descriptive Statistics

	Mean	Std. Deviation	N
Time1	8.0000	3.89872	6
Time2	10.0000	3.03315	6

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Time	Pillai's Trace	.750	15.000[a]	1.000	5.000	.012	.750
	Wilks' Lambda	.250	15.000[a]	1.000	5.000	.012	.750
	Hotelling's Trace	3.000	15.000[a]	1.000	5.000	.012	.750
	Roy's Largest Root	3.000	15.000[a]	1.000	5.000	.012	.750

a. Exact statistic

b. Design: Intercept
Within Subjects Design: Time

Mauchly's Test of Sphericity[b]

Measure:MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	1.000	.000	0	.	1.000	1.000	1.000

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b. Design: Intercept
Within Subjects Design: Time

Tests of Within-Subjects Effects

Measure:MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	Sphericity Assumed	12.000	1	12.000	15.000	.012	.750
	Greenhouse-Geisser	12.000	1.000	12.000	15.000	.012	.750
	Huynh-Feldt	12.000	1.000	12.000	15.000	.012	.750
	Lower-bound	12.000	1.000	12.000	15.000	.012	.750
Error(Time)	Sphericity Assumed	4.000	5	.800			
	Greenhouse-Geisser	4.000	5.000	.800			
	Huynh-Feldt	4.000	5.000	.800			
	Lower-bound	4.000	5.000	.800			

Tests of Within-Subjects Contrasts

Measure:MEASURE_1

Source	Time	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	Linear	12.000	1	12.000	15.000	.012	.750
Error(Time)	Linear	4.000	5	.800			

Tests of Between-Subjects Effects

Measure:MEASURE_1
Transformed Variable:Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	972.000	1	972.000	41.186	.001	.892
Error	118.000	5	23.600			

Estimated Marginal Means

Time

Measure:MEASURE_1

Time	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	8.000	1.592	3.909	12.091
2	10.000	1.238	6.817	13.183

When *a* has more than 2 levels…

Null Hypothesis:

Reduced Model: $D1_i = 0 + e_{i1}$ and $D2_i = 0 + e_{i2}$

Alternative Hypothesis:

Full Model: $D1_i = \mu_{D1} + e_{i1}$ and $D2_i = \mu_{D2} + e_{is}$

	T1	T2	T3
S1	2	3	5
S2	4	7	9
S3	6	8	8
S4	8	9	8
S5	10	13	15
S6	3	4	9
S7	6	9	8
S8	9	11	10

Reduced Model

D1 = T2 – T1	Predicted D1	Squared Errors for D1	D2 = T3 – T2	Predicted D2	Squared Errors for D2	Errors for D1 * Errors for D2
1		$(1-0)^2 = 1$	2		$(2-0)^2 = 4$	(1)(2) = 2
3		$(3-0)^2 = 9$	2		$(2-0)^2 = 4$	(3)(2) = 6
2		$(2-0)^2 = 4$	0		$(0-0)^2 = 0$	(2)(0) = 0
1		$(1-0)^2 = 1$	-1		$(-1-0)^2 = 1$	(1)(-1) = -1
3		$(3-0)^2 = 9$	2		$(2-0)^2 = 4$	(3)(2) = 6
1		$(1-0)^2 = 1$	5		$(5-0)^2 = 25$	(1)(5) = 5
3		$(3-0)^2 = 9$	-1		$(-1-0)^2 = 1$	(3)(-1) = -3
2		$(2-0)^2 = 4$	-1		$(-1-0)^2 = 1$	(2)(-1) = -2
Mean = 2		**E_R(D1) = 38**	Mean = 1		**E_R (D2) = 40**	**E_R(D1D2) = 13**

Full Model

D1 = T2 – T1	Predicted D1	Squared Errors for D1	D2 = T3 – T2	Predicted D2	Squared Errors for D2	Errors for D1 * Errors for D2
1		$(1-2)^2 = 1$	2		$(2-1)^2 = 1$	(-1)(1) = -1
3		$(3-2)^2 = 1$	2		$(2-1)^2 = 1$	(1)(1) = 1
2		$(2-2)^2 = 0$	0		$(0-1)^2 = 1$	(0)(-1) = 0
1		$(1-2)^2 = 1$	-1		$(-1-1)^2 = 4$	(-1)(-2) = 2
3		$(3-2)^2 = 1$	2		$(2-1)^2 = 1$	(1)(1) = 1
1		$(1-2)^2 = 1$	5		$(5-1)^2 = 16$	(-1)(4) = -4
3		$(3-2)^2 = 1$	-1		$(-1-1)^2 = 4$	(1)(-2) = -2
2		$(2-2)^2 = 0$	-1		$(-1-1)^2 = 4$	(0)(-2) = 0
Mean = 2		**E_F(D1) = 6**	Mean = 1		**E_F (D2) = 32**	**E_F(D1D2) = -3**

$$E_R = \begin{bmatrix} E_R(D1) & E_R(D1D2) \\ E_R(D2D1) & E_R(D2) \end{bmatrix} \quad \text{and} \quad E_F = \begin{bmatrix} E_F(D1) & E_F(D1D2) \\ E_F(D2D1) & E_F(D2) \end{bmatrix}$$

$$|E| = E(D1) * E(D2) - E(D1D2) * E(D2D1)$$

$df_R = n$ $df_F = n - (a - 1) = n - a + 1$

$$F = \frac{\left(|E_R| - |E_F|\right)/(df_R - df_F)}{|E_F|/df_F}$$

$df_{numerator} = a - 1$
$df_{denominator} = n - a + 1$
$F_{Crit} =$

Interpretation:

Assumptions

Normality Assumption

Univariate (Mixed-Model) Approach assumes the residuals are normally distributed.

Multivariate Approach assumes *multivariate normality* among the difference variables.

Independence

Both approaches assume the individuals are independent from one another.

Variance/Covariance Matrices for the One-way Repeated Measures ANOVA

Univariate (Mixed-model) Approach-Sphericity is Assumed

	σ_{T1}	σ_{T2}	σ_{T3}
σ_{T1}			
σ_{T2}			
σ_{T3}			

Multivariate Approach

	σ_{T1}	σ_{T2}	σ_{T3}
σ_{T1}			
σ_{T2}			
σ_{T3}			

Type I Error Rate and Power

- Both approaches control maintain adequate control of the *type I error rate.*
- The multivariate approach is somewhat less powerful when the sphericity assumption of the univariate approach is true.
- The multivariate approach is usually more powerful when the sphericity is not true.
- The multivariate approach is inadvisable and may even be mathematically impossible for small sample sizes.

Pairwise Comparisons for the One-Way Repeated Measures ANOVA

- Conduct pairwise comparisons using the *t* test for dependent samples, and
- Use the Bonferroni (Dunn) or Sidak approaches to maintain the familywise alpha to .05.

SPSS Instructions

→ Analyze →General Linear Model →Repeated Measures
Within-Subject Factor Name: TIME
Number of Levels: 3
→ Add → Define
Within-Subjects Variables(TIME):
T1(1)
T2(2)
T3(3)
→Options
Display Means for: TIME
√ Estimates of Effect Size
√ Descriptive Statistics

General Linear Model

Within-Subjects Factors

Measure:MEASURE_1

TIME	Dependent Variable
1	T1
2	T2
3	T3

Descriptive Statistics

	Mean	Std. Deviation	N
T1	6.0000	2.87849	8
T2	8.0000	3.33809	8
T3	9.0000	2.82843	8

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
TIME	Pillai's Trace	.865	19.148[a]	2.000	6.000	.002	.865
	Wilks' Lambda	.135	19.148[a]	2.000	6.000	.002	.865
	Hotelling's Trace	6.383	19.148[a]	2.000	6.000	.002	.865
	Roy's Largest Root	6.383	19.148[a]	2.000	6.000	.002	.865

a. Exact statistic
b. Design: Intercept
Within Subjects Design: TIME

Mauchly's Test of Sphericity[b]

Measure:MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
dimension1 TIME	.448	4.816	2	.090	.644	.728	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.
a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.
b. Design: Intercept
Within Subjects Design: TIME

Tests of Within-Subjects Effects

Measure:MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
TIME	Sphericity Assumed	37.333	2	18.667	11.200	.001	.615
	Greenhouse-Geisser	37.333	1.289	28.968	11.200	.006	.615
	Huynh-Feldt	37.333	1.455	25.657	11.200	.004	.615
	Lower-bound	37.333	1.000	37.333	11.200	.012	.615
Error(TIME)	Sphericity Assumed	23.333	14	1.667			
	Greenhouse-Geisser	23.333	9.022	2.586			
	Huynh-Feldt	23.333	10.186	2.291			
	Lower-bound	23.333	7.000	3.333			

Tests of Within-Subjects Contrasts

Measure:MEASURE_1

Source	TIME	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
TIME	Linear	36.000	1	36.000	15.750	.005	.692
	Quadratic	1.333	1	1.333	1.273	.296	.154
Error(TIME)	Linear	16.000	7	2.286			
	Quadratic	7.333	7	1.048			

Tests of Between-Subjects Effects

Measure:MEASURE_1
Transformed Variable:Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	1410.667	1	1410.667	58.545	.000	.893
Error	168.667	7	24.095			

Estimated Marginal Means

TIME

Measure:MEASURE_1

TIME	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	6.000	1.018	3.594	8.406
2	8.000	1.180	5.209	10.791
3	9.000	1.000	6.635	11.365

SPSS Instructions
→ Analyze →Compare Means →Paired Samples T Test
Paired Variables:
Pair 1: T1 T2
Pair 2: T1 T3
Pair 3: T2 T3

T-Test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	T1	6.0000	8	2.87849	1.01770
	T2	8.0000	8	3.33809	1.18019
Pair 2	T1	6.0000	8	2.87849	1.01770
	T3	9.0000	8	2.82843	1.00000
Pair 3	T2	8.0000	8	3.33809	1.18019
	T3	9.0000	8	2.82843	1.00000

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	T1 & T2	8	.966	.000
Pair 2	T1 & T3	8	.719	.044
Pair 3	T2 & T3	8	.772	.025

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper			
Pair 1	T1 - T2	-2.0000	.92582	.32733	-2.7740	-1.2260	-6.110	7	.000
Pair 2	T1 - T3	-3.0000	2.13809	.75593	-4.7875	-1.2125	-3.969	7	.005
Pair 3	T2 - T3	-1.0000	2.13809	.75593	-2.7875	.7875	-1.323	7	.227

Choosing an Appropriate Sample Size for the Multivariate Approach to the One-way Repeated Measures ANOVA for .80 power with alpha = .05 (excerpted from pp. 641-642)							
		d estimate of Effect Size					
		Small	Medium	Large			
Levels	Minimum Correlation	0.25	0.50	0.75	1.00	1.25	1.50
2	0	254	65	30	18	13	10
	.1	228	59	28	17	12	9
	.2	203	53	25	15	11	8
	.3	178	46	22	14	10	8
	.4	153	40	19	12	9	7
	.5	128	34	16	10	8	6
	.6	103	28	14	9	7	6
	.7	78	21	11	7	6	5
	.8	53	15	8	6	5	4
	.9	28	9	6	4	4	3
3	0	312	81	38	23	16	12
	.1	281	73	34	21	15	11
	.2	250	65	31	19	14	11
	.3	219	58	28	17	12	10
	.4	188	50	24	15	11	9
	.5	158	42	21	13	10	8
	.6	127	34	17	11	9	7
	.7	96	27	14	10	8	6
	.8	65	19	11	8	6	6
	.9	34	11	7	6	5	5
4	0	353	92	43	26	19	14
	.1	318	83	39	24	17	13
	.2	284	74	36	22	16	13
	.3	249	66	32	20	15	12
	.4	214	57	28	18	13	11
	.5	179	48	24	16	12	10
	.6	144	39	20	13	10	9
	.7	109	31	16	11	9	8
	.8	74	22	13	9	8	7
	.9	39	13	9	7	6	6
5	0	387	101	48	29	21	16
	.1	349	91	44	27	19	15
	.2	311	82	39	25	18	14
	.3	273	72	35	22	16	13
	.4	234	63	31	20	15	12
	.5	196	53	27	18	13	11
	.6	158	44	23	15	12	10
	.7	120	34	18	13	10	9
	.8	82	25	14	11	9	8
	.9	44	15	10	8	8	7