Trend Analysis
Maxwell & Delaney

ANOVA models are applied to data that have

- *Qualitative* independent variables
  - Gender: female, male
  - Religious affiliation: protestant, catholic, jewish, muslim, etc.

- Quantitative dependent variable

Trend analysis (a.k.a., Method of Orthogonal Polynomials) is a special case of the ANOVA models for data that have

- *Quantitative* independent variables with a limited number of values
  - Study Time: 0 hours, 1 hour, 2 hours
  - Alcohol: None, 2 drinks, 4 drinks

- Quantitative dependent variable

Regression models are applied to data that have

- *Quantitative* predictors with several values
  - Study time: ranges from 0 to 10 hours
  - Alcohol: ranges from 0 to 25 drinks

- Quantitative dependent variable

What kinds of trends are there?

Linear

Linear & Quadratic-Version 1

Quadratic

Linear & Quadratic-Version 2

Cubic

Linear & Cubic

Which test(s) of trend are possible?

Which test(s) of trend are recommended?

It is necessary to test higher-order trends regardless of whether the linear trend is statistically significant.

Oneway

Descriptives

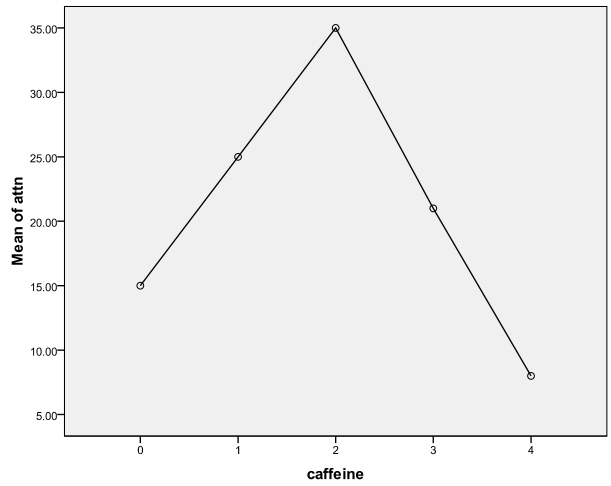
Attention Span

Caffeine Drinks	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
0	15	15.0000	8.00000	2.06559	10.5697	19.4303	5.20	29.40
1	15	25.0000	12.00000	3.09839	18.3546	31.6454	6.02	44.76
2	15	35.0000	15.00000	3.87298	26.6933	43.3067	3.26	67.20
3	15	21.0000	11.00000	2.84019	14.9084	27.0916	-.86	36.02
4	15	8.0000	7.00000	1.80739	4.1235	11.8765	-1.84	18.51
Total	75	20.8000	14.09140	1.62713	17.5579	24.0421	-1.84	67.20

ANOVA

attn

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	6252.000	4	1563.000	12.960	.000
Within Groups	8442.000	70	120.600		
Total	14694.000	74			



Trend Analysis via Complex Contrasts (Assuming Equal Population Variances Among All Groups)

1. Write the Null and Alternative Hypotheses.

   Null:

   Alternative:

2. Obtain the appropriate coefficients from the Table of Orthogonal Polynomials.

3. Determine ψ by placing the coefficients adjacent to the ordered values of the independent variable.

After step 3, you should have the Null Hypothesis in the form of a ψ contrast. The coefficients (C_j) are the numbers adjacent to the population means.

4. Calculate a sample-based version of ψ using the sample means.

   $\hat{\psi} =$

5. Test the Null Hypothesis (ψ = 0).

$$F_{\psi} = \frac{\hat{\psi}^2}{\frac{C_1^2}{n_1}MSE + \frac{C_2^2}{n_2}MSE + ... + \frac{C_a^2}{n_a}MSE} = \frac{\hat{\psi}^2}{\frac{MSE}{n}\sum_{j=1}^{a} C_j^2}$$

$$df_{Denominator} = N - a$$

6. Compare the *F* calculated in step 5 to an appropriate critical value to determine significance (choices include: Bonferroni, Sidak, Scheffe).

7. Write a sentence to communicate the results.
   - There was not a significant linear relationship between the average attention span and the amount of caffeine consumed, $F = 2.11$, $MSE = 3.12$, $p > alpha$.
   - There was a significant linear relationship between the average attention span and the amount of caffeine consumed, $F = 8.11$, $MSE = 1.27$, $p < alpha$.

Trend Analysis via Complex Contrasts (Assuming Equal Population Variances Among All Groups)

1. Write the Null and Alternative Hypotheses.

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$$F_{\psi} = \frac{\hat{\psi}^2}{\frac{C_1^2}{n_1}MSE + \frac{C_2^2}{n_2}MSE + ... + \frac{C_a^2}{n_a}MSE} = \frac{\hat{\psi}^2}{\frac{MSE}{n}\sum_{j=1}^{a} C_j^2}$$

$$df_{Denominator} = N - a$$

6. Compare the *F* calculated in step 5 to an appropriate critical value to determine significance. (choices include: Bonferroni, Sidak, Scheffe).

7. Write a sentence to communicate the results.
   - There was not a significant quadratic relationship between the average attention span and the amount of caffeine consumed, $F = 2.11$, $MSE = 3.12$, $p > alpha$.
   - There was a significant quadratic relationship between the average attention span and the amount of caffeine consumed, $F = 8.11$, $MSE = 1.27$, $p < alpha$.

Trend Analysis via Complex Contrasts (Assuming Equal Population Variances Among All Groups)

1. Write the Null and Alternative Hypotheses.

   Null:

   Alternative:

2. Obtain the appropriate coefficients from the Table of Orthogonal Polynomials.

3. Determine ψ by placing the coefficients adjacent to the ordered values of the independent variable.

After step 3, you should have the Null Hypothesis in the form of a ψ contrast. The coefficients (C_j) are the numbers adjacent to the population means.

4. Calculate a sample-based version of ψ using the sample means.

   $\hat{\psi} =$

5. Test the Null Hypothesis (ψ = 0).

$$F_{\psi} = \frac{\hat{\psi}^2}{\frac{C_1^2}{n_1}MSE + \frac{C_2^2}{n_2}MSE + ... + \frac{C_a^2}{n_a}MSE} = \frac{\hat{\psi}^2}{\frac{MSE}{n}\sum_{j=1}^{a} C_j^2}$$

$$df_{Denominator} = N - a$$

6. Compare the *F* calculated in step 5 to an appropriate critical value to determine significance. (choices include: Bonferroni, Sidak, Scheffe).

7. Write a sentence to communicate the results.

- There was not a significant cubic relationship between the average attention span and the amount of caffeine consumed, $F = 2.11$, $MSE = 3.12$, $p > alpha$.
- There was a significant cubic relationship between the average attention span and the amount of caffeine consumed, $F = 8.11$, $MSE = 1.27$, $p < alpha$.

Excerpted from Maxwell & Delaney's Table A.10. *Coefficients of Orthogonal Polynomials*								
Levels	Polynomial	Coefficients						$\sum_{j=1}^{a} C_j^2$
3	Linear	-1	0	1				2
	Quadratic	1	-2	1				6
4	Linear	-3	-1	1	3			20
	Quadratic	1	-1	-1	1			4
	Cubic	-1	3	-3	1			20
5	Linear	-2	-1	0	1	2		10
	Quadratic	2	-1	-2	-1	2		14
	Cubic	-1	2	0	-2	1		10
	Quartic	1	-4	6	-4	1		70
6	Linear	-5	-3	-1	1	3	5	70
	Quadratic	5	-1	-4	-4	-1	5	84
	Cubic	-5	7	4	-4	-7	5	180
	Quartic	1	-3	2	2	-3	1	28
	Quintic	-1	5	-10	10	-5	1	252

Maxwell & Delaney's table has coefficients for up to 10 levels of the independent variable.

SPSS Instructions
→Analyze→Compare Means→One-Way ANOVA
Dependent List: ATTENTION
Factor: CAFFEINE
→Options
√ Descriptives
√ Means Plot
→Contrast
√ Polynomial
Degree: Cubic

ANOVA

attn

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		6252.000	4	1563.000	12.960	.000
	Linear Term	Contrast	486.000	1	486.000	4.030	.049
		Deviation	5766.000	3	1922.000	15.937	.000
	Quadratic Term	Contrast	5250.000	1	5250.000	43.532	.000
		Deviation	516.000	2	258.000	2.139	.125
	Cubic Term	Contrast	1.500	1	1.500	.012	.912
		Deviation	514.500	1	514.500	4.266	.043
Within Groups			8442.000	70	120.600		
Total			14694.000	74			

Twelve volunteers agreed to experience sleep deprivation for 24, 36, or 48 hours before having their attention spans evaluated. Their data are shown below.

Descriptives

Attention Span

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
24 Hours	4	15.0000	4.00000	2.00000	8.6351	21.3649	9.12	17.52
36 Hours	4	8.0000	3.00000	1.50000	3.2263	12.7737	3.83	10.78
48 Hours	4	4.0000	1.00000	.50000	2.4088	5.5912	3.13	4.87
Total	12	9.0000	5.44393	1.57153	5.5411	12.4589	3.13	17.52

Trend Analysis via Complex Contrasts (Equal Population Variances Not Assumed)

1. Write the Null Hypothesis.

2. Obtain the appropriate coefficients from the Table of Orthogonal Polynomials. Determine ψ by placing the coefficients adjacent to the ordered values of the independent variable.

3. Calculate a sample-based version of ψ using the sample means.

$$\hat{\psi} =$$

4. Test the Null Hypothesis (ψ = 0).

$$df_{\text{Denominator}} = \frac{\left[\frac{C_1^2}{n_1}S_1^2 + \ldots + \frac{C_a^2}{n_a}S_a^2\right]^2}{\frac{\left[\frac{C_1^2}{n_1}S_1^2\right]^2}{n_1 - 1} + \ldots + \frac{\left[\frac{C_a^2}{n_a}S_a^2\right]^2}{n_a - 1}}$$

$$F_{\psi} = \frac{\hat{\psi}^2}{\frac{C_1^2}{n_1}S_1^2 + \frac{C_2^2}{n_2}S_2^2 + \ldots + \frac{C_a^2}{n_a}S_a^2}$$

5. Compare the *F* calculated in step 5 to an appropriate critical value to determine significance. Choices include Bonferroni, Sidak, and Brown-Forsythe.

6. Report the analysis in the APA results section on the next page.

Trend Analysis via Complex Contrasts (Equal Population Variances Not Assumed)

1. Write the Null Hypothesis.

2. Obtain the appropriate coefficients from the Table of Orthogonal Polynomials. Determine ψ by placing the coefficients adjacent to the ordered values of the independent variable.

3. Calculate a sample-based version of ψ using the sample means.

$$\hat{\psi} =$$

4. Test the Null Hypothesis ($\psi = 0$).

$$df_{\text{Denominator}} = \frac{\left[\frac{C_1^2}{n_1}S_1^2 + \ldots + \frac{C_a^2}{n_a}S_a^2\right]^2}{\frac{\left[\frac{C_1^2}{n_1}S_1^2\right]^2}{n_1 - 1} + \ldots + \frac{\left[\frac{C_a^2}{n_a}S_a^2\right]^2}{n_a - 1}}$$

$$F_{\psi} = \frac{\hat{\psi}^2}{\frac{C_1^2}{n_1}S_1^2 + \frac{C_2^2}{n_2}S_2^2 + \ldots + \frac{C_a^2}{n_a}S_a^2}$$

5. Compare the *F* calculated in step 5 to an appropriate critical value to determine significance. Choices include Bonferroni, Sidak, and Brown-Forsythe.

6. Complete the APA interpretation of the results.

Descriptive statistics for attention span are shown in Table 1. See Figure 1 for the relationship of attention span to sleep deprivation. A familywise alpha of .05 was used. The Welch ANOVA indicated deprivation (24, 36, 48 hours) was related to attention span, F (2, 4.6) = 14.41, p= .01, ω^2 = .69.

SPSS Instructions
→Analyze→Compare Means→One-Way ANOVA
Dependent List: ATTENTION
Factor: SLEEP
→Options
√ Descriptives
√ Welch
√ Means Plot
→Contrast
Coefficients: 1 →Add 0 →Add -1 →Add →NEXT
Coefficients: 1 →Add -2 →Add 1 →Add →CONTINUE

Oneway

ANOVA

Attention Span

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	248.000	2	124.000	14.308	.002
Within Groups	78.000	9	8.667		
Total	326.000	11			

Robust Tests of Equality of Means

Attention Span

	Statistic[a]	df1	df2	Sig.
Welch	14.410	2	4.603	.010

a. Asymptotically F distributed.

Contrast Coefficients

	Amount of Sleep Deprivation		
Contrast	24 Hours	36 Hours	48 Hours
1	-1	0	1
2	1	-2	1

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
Attention Span	Assume equal variances	1	-11.0000	2.08167	-5.284	9	.001
		2	3.0000	3.60555	.832	9	.427
	Does not assume equal variances	1	-11.0000	2.06155	-5.336	3.374	.009
		2	3.0000	3.64005	.824	5.426	.445