

One-Way Repeated Measures ANOVA
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Three Situations that Lead to a Repeated Measures Design

1. Each subject is observed on a different treatment conditions. The dependent variable is the same for all treatments.

	SHOCK	VERBAL FEEDBACK
S1	$Y = \# \text{ errors in word recall}$	
S2		
S3		
S4		
S5		

2. Each subject is measured on a different tests that have the same scale. The same scale is necessary for psychologically meaningful interpretations. This kind of design is also known as profile analysis.

	GRE-VERBAL	GRE-QUANTITATIVE
S1		
S2		
S3		
S4		
S5		

3. Each subject is measured at two or more different times (longitudinally).

	Time 1 (Pre-Test)	Time 2 (Post-Test)
S1	$Y = \text{Anxiety}$	
S2		
S3		
S4		
S5		

If $a = 2$ (as it is in the above examples), we can perform a **two correlated sample t (F) test**.

Univariate (Mixed-Model) Approach to Repeated Measures ANOVA

A one-way RM ANOVA is treated as a two-way ANOVA where “Subjects” is the second factor. The one-way RM ANOVA is different than a true two-way ANOVA.

- There is only one observation per cell because each subject represents a different level of the “Subjects” factor. Therefore, the A*S interaction and the within cell variability are confounded.
- “Subjects” is considered a *random* factor.

		A Factor		Subject Mean	Subject Effect
		Time 1	Time 2		
“Subjects” Factor	S1	8	10	$\bar{Y}_{S1} = 9$	0
	S2	3	6	$\bar{Y}_{S2} = 4.5$	-4.5
	S3	12	13	$\bar{Y}_{S3} = 12.5$	3.5
	S4	5	9	$\bar{Y}_{S4} = 7$	-2
	S5	7	8	$\bar{Y}_{S5} = 7.5$	-1.5
	S6	13	14	$\bar{Y}_{S6} = 13.5$	4.5
Time Effect		$\bar{Y}_{T1} = 8$ -1	$\bar{Y}_{T2} = 10$ 1	$\bar{Y} = 9$	

Null Hypothesis:

Reduced Model: $Y_{ij} = \mu + \pi_i + e_{ij}$ where $\hat{\mu} = \bar{Y}$ and $\hat{\pi}_i = \bar{Y}_i - \bar{Y}$

Predicted $\hat{Y}_{ij} = \bar{Y} + \hat{\pi}_i = \text{Grand Mean} + \text{Subject Effect} = \text{Subject Mean}$

$df_R = n(a - 1)$

Alternative Hypothesis:

Full Model: $Y_{ij} = \mu + \alpha_j + \pi_i + e_{ij}$ where $\hat{\mu} = \bar{Y}$ $\hat{\pi}_i = \bar{Y}_i - \bar{Y}$ $\hat{\alpha}_j = \bar{Y}_j - \bar{Y}$

Predicted $\hat{Y}_{ij} = \bar{Y} + \hat{\pi}_i + \hat{\alpha}_j = \text{Grand Mean} + \text{Subject Effect} + \text{Time Effect} = \text{Subject Mean} + \text{Time Effect}$

$df_F = (n - 1)(a - 1)$

Reduced Model: Predicted = Grand Mean + Subject Effect = Subject Mean					
Subject	Time	Score	Predicted	Prediction Error	Squared Prediction Error
S1	T1	8		-1	1
S2	T1	3		-1.5	2.25
S3	T1	12		-0.5	0.25
S4	T1	5		2	4
S5	T1	7		-0.5	0.25
S6	T1	13		-0.5	0.25
S1	T2	10		1	1
S2	T2	6		1.5	2.25
S3	T2	13		0.5	0.25
S4	T2	9		2	4
S5	T2	8		0.5	0.25
S6	T2	14		0.5	0.25
					$E_R = 16$

Full Model: Predicted = Grand Mean + Subject Effect + Time Effect					
= Subject Mean + Time Effect					
Subject	Time	Score	Predicted	Prediction Error	Squared Prediction Error
S1	T1	8		0	0
S2	T1	3		-0.5	0.25
S3	T1	12		0.5	0.25
S4	T1	5		-1	1
S5	T1	7		0.5	0.25
S6	T1	13		0.5	0.25
S1	T2	10		0	0
S2	T2	6		0.5	0.25
S3	T2	13		-0.5	0.25
S4	T2	9		1	1
S5	T2	8		-0.5	0.25
S6	T2	14		-0.5	0.25
					$E_F = 4$

$$F_{Time} = \frac{(E_R - E_F) / (df_R - df_F)}{E_F / df_F} =$$

$$F_{Crit} = 6.61$$

$$\text{Partial } \eta^2 \text{ for } A \text{ factor} = \frac{SS_A}{SS_A + SS_{Error}} = \frac{E_R - E_F}{E_R}$$

$$\text{Partial } \hat{\omega}^2 = \frac{(a-1)(F_A - 1)}{(a-1)(F_A - 1) + na}$$

SPSS Instructions

→ Analyze → General Linear Model → Repeated Measures
 Within-Subject Factor Name: TIME
 Number of Levels: 2
 → Add → Define
 Within-Subjects Variables(Time):
 Time1(1)
 Time2(2)
 → Options
 Display Means for: TIME
 Estimates of Effect Size
 Descriptive Statistics

General Linear Model

Within-Subjects Factors

Measure: MEASURE_1

Time	Dependent Variable
1	Time1
2	Time2

Descriptive Statistics

	Mean	Std. Deviation	N
Time1	8.0000	3.89872	6
Time2	10.0000	3.03315	6

Multivariate Tests^b

Effect	Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Time Pillai's Trace	.750	15.000 ^a	1.000	5.000	.012	.750
Wilks' Lambda	.250	15.000 ^a	1.000	5.000	.012	.750
Hotelling's Trace	3.000	15.000 ^a	1.000	5.000	.012	.750
Roy's Largest Root	3.000	15.000 ^a	1.000	5.000	.012	.750

a. Exact statistic

b. Design: Intercept

Within Subjects Design: Time

Mauchly's Test of Sphericity^b

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon ^a		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	1.000	.000	0	.	1.000	1.000	1.000

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b. Design: Intercept

Within Subjects Design: Time

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	Sphericity Assumed	12.000	1	12.000	15.000	.012	.750
	Greenhouse-Geisser	12.000	1.000	12.000	15.000	.012	.750
	Huynh-Feldt	12.000	1.000	12.000	15.000	.012	.750
	Lower-bound	12.000	1.000	12.000	15.000	.012	.750
Error(Time)	Sphericity Assumed	4.000	5	.800			
	Greenhouse-Geisser	4.000	5.000	.800			
	Huynh-Feldt	4.000	5.000	.800			
	Lower-bound	4.000	5.000	.800			

Tests of Within-Subjects Contrasts

Measure: MEASURE_1

Source	Time	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	Linear	12.000	1	12.000	15.000	.012	.750
Error(Time)	Linear	4.000	5	.800			

Tests of Between-Subjects Effects

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	972.000	1	972.000	41.186	.001	.892
Error	118.000	5	23.600			

Estimated Marginal Means

Time

Measure: MEASURE_1

Time	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	8.000	1.592	3.909	12.091
2	10.000	1.238	6.817	13.183

UNIVARIATE ONE-WAY REPEATED MEASURES ANOVA ASSUMPTIONS

1. The residuals are normally distributed.

2. The residuals are independent.

3. We have homogeneity of treatment difference variances, a.k.a. *sphericity*.

What does *sphericity* mean? Sphericity means that the population variances of the **difference scores** are equal.

Sphericity Assumption of the univariate one-way RM ANOVA	
If $a = 2$ levels...	
If $a = 3$ levels...	
If $a = 4$ levels...	

Compound Symmetry is a slightly more restrictive assumption than *Sphericity*, but it is easier to demonstrate. *Compound symmetry* requires that all the population variances be the same and that all the population covariances be the same.

An example of compound symmetry being a **valid assumption** is:

	σ_{A1}	σ_{A2}	σ_{A3}	σ_{A4}
σ_{A1}				
σ_{A2}				
σ_{A3}				
σ_{A4}				

An example of compound symmetry **not** being a **valid** assumption is:

	σ_{A1}	σ_{A2}	σ_{A3}	σ_{A4}
σ_{A1}				
σ_{A2}				
σ_{A3}				
σ_{A4}				

Realistically, the *Sphericity Assumption is NOT valid* in the real world. Unfortunately, the Repeated Measures ANOVA is NOT robust to violation of the *sphericity assumption* without modification!

We modify the degrees of freedom of the one-way repeated measures ANOVA to compensate for violation of the sphericity assumption.

$$df'_{\text{numerator}} = \varepsilon(a - 1) \quad \text{and} \quad df'_{\text{denominator}} = \varepsilon(n - 1)(a - 1)$$

where ε is a function of the unequal variances of the difference scores.

$$\frac{1}{a - 1} \leq \varepsilon \leq 1$$

$\varepsilon = 1$ when the population variances of the difference scores are all equal.

ε is less than one when the population variances of the difference scores are unequal.

This works to decrease our degrees of freedom and make the test more conservative.

We must estimate ε because it is a parameter. There are two well-known sample estimates of ε :

Box's $\hat{\varepsilon}$ (a.k.a. Greenhouse-Geiser, G-G on SAS)

Huynh-Feldt's $\tilde{\varepsilon}$ (a.k.a. H-F on SAS)

Pairwise Comparisons for the One-way Repeated Measures ANOVA

The research indicates using a pooled error term (like *MSE*) is not a good idea because of the sphericity assumption issue. Instead, the following technique is recommended:

- Conduct pairwise comparisons using the *t* test for dependent samples, and
- Use the Bonferroni (Dunn), Sidak, or Scheffe approaches to maintain the familywise alpha to .05.

SPSS Instructions

→ Analyze → General Linear Model → Repeated Measures
 Within-Subject Factor Name: WORDTYPE
 Number of Levels: 3
 → Add → Define
 Within-Subjects Variables(WORDTYPE):
 FAMILIAR(1)
 NONSENSE(2)
 UNFAMILIAR(3)
 → Options
 Display Means for: WORDTYPE
 ✓ Estimates of Effect Size
 ✓ Descriptive Statistics

General Linear Model

Within-Subjects Factors

Measure: MEASURE_1

WordType	Dependent Variable
1	Familiar
2	Nonsense
3	Unfamiliar

Descriptive Statistics

	Mean	Std. Deviation	N
Familiar	10.4375	2.75605	16
Nonsense	6.6875	3.89391	16
Unfamiliar	8.3750	3.81007	16

Multivariate Tests^b

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
WordType	Pillai's Trace	.656	13.376 ^a	2.000	14.000	.001	.656
	Wilks' Lambda	.344	13.376 ^a	2.000	14.000	.001	.656
	Hotelling's Trace	1.911	13.376 ^a	2.000	14.000	.001	.656
	Roy's Largest Root	1.911	13.376 ^a	2.000	14.000	.001	.656

a. Exact statistic

b. Design: Intercept

Within Subjects Design: WordType

Mauchly's Test of Sphericity^b

Measure:MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon ^a		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
WordType	.677	5.453	2	.065	.756	.823	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

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b. Design: Intercept

Within Subjects Design: WordType

Tests of Within-Subjects Effects

Measure:MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
WordType	Sphericity Assumed	112.875	2	56.438	11.154	.000	.426
	Greenhouse-Geisser	112.875	1.512	74.645	11.154	.001	.426
	Huynh-Feldt	112.875	1.646	68.595	11.154	.001	.426
	Lower-bound	112.875	1.000	112.875	11.154	.004	.426
Error(WordType)	Sphericity Assumed	151.792	30	5.060			
	Greenhouse-Geisser	151.792	22.682	6.692			
	Huynh-Feldt	151.792	24.683	6.150			
	Lower-bound	151.792	15.000	10.119			

Tests of Within-Subjects Contrasts

Measure:MEASURE_1

Source	WordType	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
WordType	Linear	34.031	1	34.031	15.252	.001	.504
	Quadratic	78.844	1	78.844	9.995	.006	.400
Error(WordType)	Linear	33.469	15	2.231			
	Quadratic	118.323	15	7.888			

Tests of Between-Subjects Effects

Measure:MEASURE_1

Transformed Variable:Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	3468.000	1	3468.000	127.709	.000	.895
Error	407.333	15	27.156			

Estimated Marginal Means

WordType

Measure: MEASURE_1

WordType	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	10.438	.689	8.969	11.906
2	6.688	.973	4.613	8.762
3	8.375	.953	6.345	10.405

SPSS Instructions

→ Analyze → Compare Means → Paired Samples T Test

Paired Variables:

Pair 1: Familiar Nonsense

Pair 2: Familiar Unfamiliar

Pair 3: Nonsense Unfamiliar

T-Test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Familiar	10.4375	16	2.75605	.68901
	Nonsense	6.6875	16	3.89391	.97348
Pair 2	Familiar	10.4375	16	2.75605	.68901
	Unfamiliar	8.3750	16	3.81007	.95252
Pair 3	Nonsense	6.6875	16	3.89391	.97348
	Unfamiliar	8.3750	16	3.81007	.95252

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Familiar & Nonsense	16	.498	.050
Pair 2	Familiar & Unfamiliar	16	.840	.000
Pair 3	Nonsense & Unfamiliar	16	.534	.033

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
						Lower	Upper		
Pair 1	Familiar - Nonsense	3.75000	3.47371	.86843	1.89899	5.60101	4.318	15	.001
Pair 2	Familiar - Unfamiliar	2.06250	2.11246	.52812	.93685	3.18815	3.905	15	.001
Pair 3	Nonsense - Unfamiliar	-1.68750	3.71876	.92969	-3.66909	.29409	-1.815	15	.090