MATH 2050 **NAME**_______________________________________

SAMPLE TEST 2 **SPRING 2007**

1. Professor Binary gives a True-False quiz consisting of 10 questions. Sam Serendipity did not study so he randomly and independently selected an answer for each question. Find the probability that he got all 10 correct.

$P(all\, ten\, correct) = \left(\frac{1}{2}\right)^{\mathbf{10}} = \frac{1}{1024}$

answer: $\frac{1}{1024}$

2. If a pair of fair dice is rolled, find the probability that the sum of upturned faces is 5.

$P(sum\, is\, 5) = P(\{(1,\mathbf{4}),(\mathbf{4},1),(2,\mathbf{3}),(3,2)\} = \frac{4}{36}$

answer: $\frac{4}{36}$

3. Consider the following data on traffic accidents.

age group	% of drivers	accident probability
16 to 25	21	.11
26 to 45	33	.06
46 to 65	32	.05
over 65	14	.08

Calculate the probability that a randomly chosen driver has an accident.

answer: .0701

4. Three cards are dealt from a well-shuffled deck of playing cards. Find the probability of the event that at least one of the cards is a red face card. (A deck has 12 face cards, 40 non-face cards, 26 red cards, and 26 black cards.)

$There\, are\, 6\, red\, face\, cards\, and\, 46\, non\text{-}\mathbf{red\text{-}face}\, cards.$

$$\begin{aligned} P(at\, least\, one\, red\, face\, card) &= 1 - P(no\, red\, face\, cards) \\ &= 1 - \tfrac{46}{52}\cdot\tfrac{45}{51}\cdot\tfrac{44}{50} \\ &= .3131 \end{aligned}$$

answer: .3131

5. There are 75 people at a party, all of whom are musicians or writers. Fifty of the attendees are musicians. Forty of the attendees are writers. If one of the attendees is randomly selected to sing, what is the probability that the singer is a musician but not a writer.

Since 50 $+ 40 -$ 75 $= 15$, there must be 15 musician-writers, and hence 35 musician-nonwriters. Thus $P(\textit{musician-nonwriter}) = 35/75$.

answer: $7/15 \approx .4667$

6. Coach Ficklepick must choose 4 players from her 10 reserve players to play in Saturday's game. How many combinations are possible?

$\binom{10}{4} = 210$ **answer:** 210

7. If the odds for event E are 5 to 3, then the probability of event E is ...

$P(E) = \frac{5}{5+3} = \frac{5}{8}$

answer: $\frac{5}{8}$

8. Deal one card from a well-shuffled deck of playing cards. Let R denote the event that the card is a red card, and let J be the event that the card is a jack. Which of the following statements is **false**?
$HMMMMM$...**????**

A) Events R and J are mutually exclusive
B) Events R and J are independent.
C) $P(R|J) = P(R)$
D) $P(R \text{ and } J) = P(R) \cdot P(J)$
E) $P(J) < P(R)$

9. Suppose the probability that a jet pilot is shot down during any mission is .02. Assume that the outcomes of all missions are mutually independent. Find the maximum number of missions the pilot can fly such that the probability of never being shot down is at least .90.

answer: 5

10. A barrel contains 20 apples of which 2 are rotten. If 5 apples are randomly picked from the barrel, what is the probability that none of the 5 apples picked are rotten. Round your answer to 4 decimal places.

answer: $.5526$

$\boxed{11}$. A geologist is using seismographs to test for oil. It is found that if oil is present, the test gives a positive result 90% of the time, and if the oil is not present, the test gives a positive result 5% of the time. Oil is actually present in 2% of the cases tested. If the test shows positive, what is the probability that oil is present?

$$P(O|+) = \frac{P(+|O)P(O)}{P(+)} = \frac{(.90)(.02)}{(.90)(.02)+(.05)(.98)} \approx .269$$

answer: .269

$\boxed{12}$. A local trade union consists of 75 plumbers and 75 electricians. Classified according to rank:

	Apprentice	Journeyman	Master	Totals
Plumbers	25	20	30	75
Electricians	15	40	20	75
Totals	40	60	50	

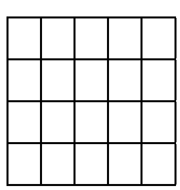
A member of the union is selected at random. Given that the person selected is a journeyman, find the probability that she is a plumber.

$$P(P|J) = \frac{20}{60}$$

answer: $\frac{1}{3}$

$\boxed{13}$. You live 4 blocks south and 5 blocks east of Granny's Grocery. You randomly select one of the many direct routes and proceed to walk to Granny's. Unbeknownst to you a group of hoodlums is harassing pedestrians at the intersection 3 blocks north and 1 block west of your home. What is the probability that your route will **not** go through the hoodlum intersection.? Round your answer to 3 decimal places.

$$P(not\, cross\, paths) = \frac{\binom{9}{4}-\binom{5}{4}\binom{4}{1}}{\binom{9}{5}} = \frac{126-20}{126} = \frac{106}{126} \approx .8413$$



answer: .8413

$\boxed{14}$. The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 80,000 miles and a standard deviation of 5000 miles. What is the probability a particular tire of this brand will last longer than 70,000 miles?
normalcdf(70000,10^99,80000,5000) $= .97725$

answer: $.97725$

$\boxed{15}$. The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 80,000 miles and a standard deviation of 5000 miles. What warranty should the company use if they want 95% of the tires to outlast the warranty? (Round to the next lower 1000.)
invNorm(.05,8000,5000)=71775.73187

answer:71,000

$\boxed{16}$. According to a recent study, 2 in every 3 men have cheated at some point in their lives. Suppose we have randomly and independently sampled twenty-five men and asked whether they have cheated at some point in their lives. If X denotes the number of the 25 men who have cheated, find the mean of X.

$\mu = np = 25(2/3) = 16\frac{2}{3}$ **answer:** 16.667

$\boxed{17}$. According to a recent study, 2 in every 3 men have cheated at some point in their lives. Suppose we have randomly and independently sampled twenty-five men and asked whether they have cheated at some point in their lives. If X denotes the number of the 25 men who have cheated, find the standard deviation of X.

$\sigma = \sqrt{np(1-p)} = \sqrt{25(2/3)(1/3)} = \sqrt{50/9} \approx 2.357$ **answer:** 2.357

$\boxed{18}$. What is the mean of the following probability distribution?

x	0	1	2	3	4
$f(x)$	0.3	0.3	0.2	0.1	0.1

answer: 1.4

$\boxed{19}$. What is the standard deviation (accurate to 3 decimal places) of the following probability distribution?

x	0	1	2	3	4
$f(x)$	0.3	0.3	0.2	0.1	0.1

$E(X^2) = .3 + .8 + .9 + 1.6 = 3.6$
$\sigma = \sqrt{3.6 - 1.4^2} = \sqrt{1.64}$

answer: $\sqrt{1.64} \approx 1.281$

20.Suppose X is a binomial random variable with p = 0.70 and n = 1200. Use the normal approximation (with a continuity correction) to find the approximate value of $P(X \leq 825)$.

$normalcdf(-10\hat{}99, 825.5,\ 1200*.70, (1200*.7*.3)\hat{}.5) = .1805$

answer: .1805

21. You pay x dollars to play the following game. You randomly pick a card from a deck of cards. If the card is an ace the "house" pays you \$30. If the card is a face card, the house pays you \$7. If the card value is 2 through 10, the house pays you \$3. How much should you pay to play in order to make the game "fair"?

$E(house\, payout\, \$) = \frac{4(30)+\mathbf{12(7)+36(3)}}{\mathbf{52}} = 6$

answer: \$6.00

22. Suppose the birth weights of newborn babies at City General Hospital have a normal distribution with mean 7.4 lb. and standard deviation 1.3 lb. If 100 birth weights are randomly selected and $\overline{X}$ denotes the sample mean weight, find $P(7 \leq \overline{X} \leq 7.5)$.

$normalcdf(7, 7.5, 7.4, 1.3/10) = .7781$

answer: .7781

23. Nine students in one large class were all born in the month of September. Find the probability that **none** of the nine have the same birthday.

$\frac{{}_{30}P_9}{30^9} = .26377$

answer: .26377

24. A human gene carries a certain disease from the mother to the child with a probability rate of 25%. That is, there is a 25% chance that the child becomes infected with the disease. Suppose a female carrier of the gene has six children. Assume that the infections of the six children are independent of one another. Find the probability that exactly two of the children get the disease from their mother. (Round your answer to 4 decimal places.)

$\binom{6}{2}(.25)^2(.75)^4 = binompdf(6, .25, 2) = .2966$ **answer:** .2966

25. In a group of students, 20% tease other students, 25% talk in class, and 15% do both. What is the probability that a randomly selected student from the group has at least one of these bad habits?

$$.20 + .25 - .15 = .30$$

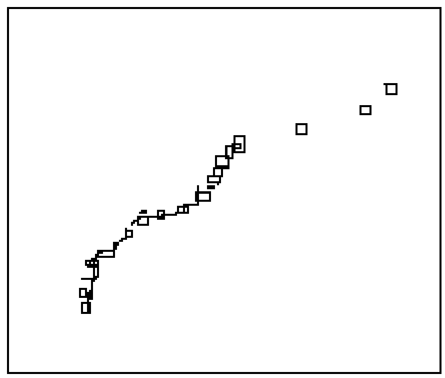
answer: .30

26. Below is a set of random data (in ascending order).

12, 13, 13, 13, 14, 17, 17, 19, 19, 19, 20, 21, 21, 21, 21, 22, 25,
25, 29, 29, 30, 32, 33, 36, 36, 36, 36, 36, 36, 37, 38, 48, 59, 63

Use a TI-83 calculator to construct a normal probability plot of the data.

Rough sketch of normal probability plot:



Does the plot above indicate that the population sampled could be normally distributed? Yes **No**

27. Which of the following conditions is not necessary for a binomial experiment?
HMMMMMM?????

a. There are a fixed number n of trials.
b. For each trial, the results can be broken down into exactly two outcomes: "success" and "failure".
c. The trials are independent.
d. The probability of "success" equals the probability of "failure".
e. A "success" on one trial does not change the probability of "success" on another trial.

28. A psychic network received telephone calls last year from over 1.5 million people. One of the psychic network's psychics agreed to take part in the following experiment. Four different cards are shuffled, and one is chosen at random. The psychic will then try to identify which card was drawn without seeing it. Assume that the experiment was repeated 10 times and that the results of any two experiments are independent of one another. If we assume that the psychic is a fake (i.e., they are merely guessing at the cards and have no psychic powers), find the probability that they 3 or more correctly.

1 − **binomcdf(10,.25,2)** = .474407196

answer: .4744