NOTES ON CHAPTER 5
MATRIX APPROACH TO SIMPLE LINEAR REGRESSION

I. Basic Properties of Matrices

- definition of matrix and of vector
- row/column subscript notation
- transpose of a matrix
- matrix addition and subtraction
- matrix multiplication: by scalar; by matrix.
- special types of matrices: square, symmetric, diagonal, identity, scalar, nonsingular.
- vectors and matrices consisting of ones; the zero vector and matrix.
- linear dependence and rank of a matrix
- inverse of a nonsingular matrix; determinant of a square matrix
- some basic theorems on p 194
- quadratic forms p 206-207

II. Random Matrices and Vectors

- definition of a random matrix (random vector)
- expectation of random matrix
- variance-covariance matrix of a random vector
- theorems for $\mathbf{W} = \mathbf{AY}$ where $\mathbf{A}$ is a matrix of constants, $\mathbf{Y}$ is a random vector (p 197)

III. Simple Linear Regression Analysis in Matrix Format

- regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$ where random error vector ϵ consists of independent normal rv's

  with $\mathbf{E\{\epsilon\} = 0}$ and $\sigma^2\{\epsilon\} = \sigma^2\mathbf{I}$.

- regression function $\mathbf{E\{Y\} = X}\beta$

- normal equations $\mathbf{X'Xb = X'Y}$ and solution $\mathbf{b = (X'X)^{-1}X'Y}$

- fitted vector $\hat{\mathbf{Y}} = \mathbf{Xb}$ or $\hat{\mathbf{Y}} = \mathbf{HY}$ where $\mathbf{H} = \mathbf{X(X'X)^{-1}X'}$, the hat matrix

- residual vector $\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}}$

- sums of squares as quadratic forms ($\mathbf{Y'AY}$ form)

  SSTO $= \mathbf{Y'(I - \frac{1}{n}\ J)Y}$, SSE $= \mathbf{Y'(I - H)Y}$, SSR $= \mathbf{Y'(H - \frac{1}{n}\ J)Y}$

- varaince-covariance matrices: $\sigma^2\{\mathbf{b}\} = \sigma^2\mathbf{(X'X)^{-1}}$ and $s^2\{\mathbf{b}\} = \text{MSE}\mathbf{(X'X)^{-1}}$;

  $\sigma^2\{\hat{Y}_h\} = \mathbf{X}_h'\sigma^2\{\mathbf{b}\}\mathbf{X}_h$ where $\mathbf{X}_h' = [\,1 \quad X_h\,]$, $s^2\{\hat{Y}_h\} = \mathbf{X}_h' s^2\{\mathbf{b}\}\mathbf{X}_h$, and $\hat{Y}_h = \mathbf{X}_h'\mathbf{b}$;

  $s^2\{Y_{h(new)}\} = \text{MSE} + s^2\{\hat{Y}_h\}$.