

**Brand preference.** In a small-scale study of the relation between degree of brand liking ( $Y$ ) and moisture content ( $X_1$ ) and sweetness ( $X_2$ ) of the product, the following results were obtained (data are coded):

i:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$X_{i1}$ :	4	4	4	4	6	6	6	6	8	8	8	8	10	10	10	10
$X_{i2}$ :	2	4	2	4	2	4	2	4	2	4	2	4	2	4	2	4
$Y_i$ :	64	73	61	76	72	80	71	83	83	89	86	93	88	95	94	100

Assume that the (additive) regression model (7.1) with independent normal error terms is appropriate.

- a. Find the estimated regression coefficients.  $\mathbf{b}' = [ \quad \quad \quad ]$

State the estimated regression function.  $\hat{Y} =$

Interpret  $b_1$ :

- b. Test whether there is a regression relation using a level of significance of .01 .

$H_0$ :

$H_a$ :

Decision rule:

Test statistic:

P-value and conclusion:

c. Estimate  $\beta_1$  and  $\beta_2$  jointly by the Bonferroni procedure using 99% family confidence level.

$$s^2\{\mathbf{b}\} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Hence  $s\{b_1\} =$  \_\_\_\_\_ and  $s\{b_2\} =$  \_\_\_\_\_

Also  $t(1 - \frac{\alpha}{2g}, n - p) =$  \_\_\_\_\_

Therefore

$$\leq \beta_1 \leq$$

$$\leq \beta_2 \leq$$

d. Coefficient of multiple determination:  $R^2 =$  \_\_\_\_\_

Interpret  $R^2$ :

e. Find coefficient of simple determination between  $Y_i$  and  $\hat{Y}_i$ :  $r_{Y\hat{Y}}^2 =$  \_\_\_\_\_

f. Obtain an interval estimate of the mean response  $E\{Y_h\}$  when  $X_{h1} = 5$  and  $X_{h2} = 4$ . Use a 99 percent confidence coefficient.

point estimate:  $\hat{Y}_h =$  \_\_\_\_\_  $s\{\hat{Y}_h\} =$  \_\_\_\_\_

$t(1 - \alpha/2, n - p) =$  \_\_\_\_\_  $\leq E\{Y_h\} \leq$  \_\_\_\_\_

Interpret your interval estimate (in plain English):

g. Obtain a prediction interval for a new observation  $Y_{h(new)}$  when  $X_{h1} = 5$  and  $X_{h2} = 4$ . Use a 99 percent confidence coefficient.

point estimate:  $\hat{Y}_h =$  \_\_\_\_\_  $s\{Y_{h(new)}\} =$  \_\_\_\_\_

$t(1 - \alpha/2, n - p) =$  \_\_\_\_\_  $\leq Y_{h(new)} \leq$  \_\_\_\_\_

h. Obtain the residuals. (Do not list them here.)

i. Plot the residuals against  $\hat{Y}$ ,  $X_1$ , and  $X_2$  on separate graphs. Also prepare a normal probability plot. Print the residuals and do the above plots on a separate sheet.

$$\text{Corr}(e_{(i)}, E\{e_{(i)}\}) =$$

Analysis of plots and summary of findings:

j. Lack of fit test.  $H_0$ :

$H_a$ :

Decision rule:

Conclusion: