CHAPTER 8

P0LYNOMIAL REGRESSION

- Used when data indicates a curvilnear response function.
- Extrapolation beyond the range of data is not recommended.
- Example. Second order model in one independent variable:

$$Y_i = \beta_0 + \beta_1 x_i + \beta_{11} x_i^2 + \epsilon_i$$

with $x_i = X_i - \bar{X}$ so as to reduce multicollinearity and avoid computational difficulties.

- Example. Second order model with two independent variables:

$$\mathrm{Y_i} = \beta_0 + \beta_1 x_{\mathrm{i1}} + \beta_2 x_{\mathrm{i2}} + \beta_{11} x_{\mathrm{i1}}^2 + \beta_{22} x_{\mathrm{i2}}^2 + \beta_{12} x_{\mathrm{i1}} x_{\mathrm{i2}} + \epsilon_{\mathrm{i}}$$

with $x_{\mathrm{i1}} = X_{\mathrm{i1}} - \bar{X}_1$ and $x_{\mathrm{i2}} = X_{\mathrm{i2}} - \bar{X}_2$.
The coefficients β_1 and β_2 are *linear effect* coefficients,
β_{11} and β_{22} are *quadratic effect* coefficients, and
β_{12} is the *interaction effect* coefficient for the cross product term.
Response surface is three-dimensional. See examples and figures in text.

- For an example of a third order model see text. Models of oder higher than the third are seldom used. Often one fits a second or third order model and then explores whether a lower order is adequate.
- Hierarchical approach to fitting polynomial model: fit the higher order model first, then explore whether a lower-order model is adequate.

QUALITATIVE PREDICTORS

Example. If gender is a second predictor variable in a model, one may use indicator variable

$$X_2 = \begin{cases} 1 \text{ if person is female} \\ 0 \text{ if person is male} \end{cases}$$

If a qualitative predictor variable has k categories, then $k-1$ indicator variables are needed.

Example. Suppose a second predictor variable has categories urban, suburban, and rural.

The one may use $X_2 = \begin{cases} 1 \text{ if urban} \\ 0 \text{ otherwise} \end{cases}$

$$X_3 = \begin{cases} 1 \text{ if suburban} \\ 0 \text{ otherwise} \end{cases}$$

If a model consists of only qualitative predictor variables, it is an *analysis of variance* model.