Equivalence of the t-test and F-test in simple linear regression

In mathematical statistics, a common exercise shows that if $T \sim t(\nu)$, then $T^2 \sim F(1, \nu)$.

Now we will show, for a simple linear regression model, the square of the t-test statistic for a significant regression relationship is equal to the F-test statistic, that is, $\left(\frac{b_1}{s\{b_1\}}\right)^2 = \frac{MSR}{MSE}$.

We first show that $SSR = b_1^2 SSXX$.

$$SSR = \sum(\widehat{Y}_i - \overline{Y})^2$$

$$= \sum(b_0 + b_1 X_i - \overline{Y})^2 \qquad [\text{since} \widehat{Y}_i = b_0 + b_1 X_i]$$

$$= \sum(\overline{Y} - b_1 \overline{X} + b_1 X_i - \overline{Y})^2 \qquad [\text{since } b_0 = \overline{Y} - b_1 \overline{X}]$$

$$= b_1^2 \sum(X_i - \overline{X})^2$$

$$= b_1^2 SSX.$$

Therefore, since $b_1^2 = SSR/SSX$ and $s^2\{b_1\} = MSE/MSR,$ we have

$$\left(\frac{b_1}{s\{b_1\}}\right)^2 = \frac{SSR/SSX}{MSE/SSX}$$

$$= \frac{SSR}{MSE}$$

$$= \frac{MSR}{MSE}.$$