# An Identity for SSxx

$$\boxed{\mathbf{SSxx} = \frac{1}{n} \sum_{i<j} (x_i - x_j)^2}$$

*Proof.*

$$\begin{aligned}
\sum_{i<j}(x_i - x_j)^2 &= \sum_{i<j}(x_i^2 + x_j^2 - 2x_i x_j) \\
&= (n-1)\sum_{i=1}^{n} x_i^2 - \sum_{i<j} 2x_i x_j \\
&= (n-1)\sum_{i=1}^{n} x_i^2 - \left(\left(\sum_{i=1}^{n} x_i\right)^2 - \sum_{i=1}^{n} x_i^2\right) \\
&= n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \\
&= n\left(\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 / n\right) \\
&= n\left(\sum_{i=1}^{n} x_i^2 - n\overline{x}^{\,2}\right) \\
&= n\left(\sum_{i=1}^{n} x_i^2 - 2n\overline{x}^{\,2} + n\overline{x}^{\,2}\right) \\
&= n\left(\sum_{i=1}^{n} x_i^2 - 2\overline{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \overline{x}^{\,2}\right) \\
&= n\sum_{i=1}^{n}(x_i - \overline{x})^2 \\
&= n\,SSxx \qquad \square
\end{aligned}$$