

Hypothesis Test Terminology

- **null hypothesis, H_0 :** the hypothesis that is initially assumed to be true.
If the null hypothesis involves the value of a parameter θ , it typically takes one of the following forms: $\theta = \theta_0$, $\theta \geq \theta_0$, or $\theta \leq \theta_0$. Note that equal sign is in all the forms. The specified value θ_0 is called the null value.
- **alternative hypothesis, H_1 :** the hypothesis the researcher or investigator is attempting to infer is true. To reject the null hypothesis and thus conclude that the alternative hypothesis is the better hypothesis, one needs a lot of evidence that points towards the alternative hypothesis. The amount of evidence required depends on the pre-assigned level of significance α .
- **conclusion of a hypothesis test:** either (i) reject H_0 (and thus conclude H_a)
or (ii) do not reject H_0
Typically, one needs a lot of evidence contradictory to H_0 in order to reject H_0 .
- **Type I error:** the error that occurs when one rejects H_0 when in fact H_0 is true
- **Type II error:** the error that occurs when one does not reject H_0
when in fact H_0 is false
- **level of significance (α):** the maximum probability of making a Type I error
- **beta (β):** the probability of making a Type II error
- **test statistic:** a summary of the evidence obtained from the random sample.
For example, the test statistic for a large sample test of μ is given by $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$.
If the test statistic is sufficiently extreme relative to the null hypothesis, the null hypothesis is rejected.
- **decision rule:** the rule that specifies when one is to reject the null hypothesis
- **critical value:** the boundary value (based on the sampling distribution of the test statistic and the level of significance) that is used in the decision rule.
If the test statistic is more extreme than the critical value, the null hypothesis is rejected.
- **critical region:** if the test statistic falls in the critical region(s), reject H_0
- **power of the test :** $P(\text{reject } H_0 \text{ given that } H_0 \text{ is false}) = 1 - \beta$
- **p-value** (observed significance level): the probability under H_0 of observing a value of the test statistic that is at least as contradictory to the null hypothesis as the actual one computed from the sample data.

- **ANALOGIES**

Criminal Justice	Hypothesis Testing
claim defendant is innocent	null hypothesis H_o
claim defendant is guilty	alternative hypothesis H_a
“innocent until proven guilty”	null hypothesis is initially accepted
jury returns a guilty verdict	null hypothesis is rejected
jury declares defendant not guilty	null hypothesis is not rejected
evidence presented in court	value of the test statistic
evidence beyond reasonable doubt	test statistic in critical region
innocent person is declared “guilty”	Type I error
guilty person is found “not guilty”	Type II error
max. prob. of convicting innocent person	level of significance α
chance of obtaining evidence as damaging as that presented in court given that the defendant is innocent	p -value