STAT 6604 **Experimental Design**
Test 2 **Name___________KEY______________**

Show all pertinent work. Be neat.

Part I (Each correct answer in problems 1 – 3 is worth 4 points.)

Suppose we have the following estimated treatment means in a two-factor (fixed levels) study with equal sample sizes ($n = 5$).

		Factor B 1	**Factor B** 2
	1	30	50
Factor A	**2**	95	115
	3	55	75

1. Is there significant evidence of interaction? Justify your answer.

There is no evidence of significant interaction since

$$\overline{y}_{21} - \overline{y}_{11} = \overline{y}_{22} - \overline{y}_{12} \text{ and } \overline{y}_{31} - \overline{y}_{21} = \overline{y}_{32} - \overline{y}_{22}$$
$$95 - 30 = 115 - 50 \text{ and } 55 - 95 = 75 - 115.$$

Analogously, a plot of treatment means vs. factor A levels (with lines connecting factor B levels) shows parallel lines.

2. Specify two sums of squares which **cannot** be obtained from the information in the table above.

(Circle two:) [SSTO] SSTR [SSE] SSA SSB SSAB

3. Provide unbiased point estimates for the following parameters.

parameter	estimate
$\mu_{..}$	$\overline{y}_{...} = 70$
μ_{12}	$\overline{y}_{12.} = 50$
μ_{31}	$\overline{y}_{31.} = 55$
$\mu_{.2}$	$\overline{y}_{.2.} = \frac{50+115+75}{3} = 80$
$\mu_{3.}$	$\overline{y}_{3..} = \frac{55+75}{2} = 65$
α_1	$\overline{y}_{1..} - \overline{y}_{...} = 40 - 70 = -30$
β_2	$\overline{y}_{.2.} - \overline{y}_{...} = 80 - 70 = 10$
$(\alpha\beta)_{12}$	$\overline{y}_{...} + \overline{y}_{12.} - \overline{y}_{.2.} - \overline{y}_{1..} = 70 + 50 - 80 - 40 = 0$

Part I (continuation...)

4. (10 points). Complete the following ANOVA table based on the earlier information and the additional information contained in the table.

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ANALYSIS OF VARIANCE  response
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SOURCE	DF	SS	MS
TREATMENT	5	24500	4900
factAlvl	2	21500	10750
factBlvl	1	3000	3000
INTERACTION	2	0	0
ERROR	24	2446	102
TOTAL	29	26946	

5. (20 points.) Pairwise comparisons of factor level means are desired with family confidence coefficient of .90. Use either (i) the Bonferroni method directly (with g = 4);
or (ii) the Bonferroni method in conjunction with the Tukey method.

(i) Bonferroni critical value. Since $1 - .10/(2*4) = .9875$, we use $t(.9875, 24) = 2.39095$.

Margin of errors. $E = t\sqrt{MSE(\frac{2}{bn})} = 2.39095\sqrt{102\left(\frac{2}{10}\right)} = 10.799$ **(for fac. A comparisons)**

$E = t\sqrt{MSE(\frac{2}{an})} = 2.39095\sqrt{102\left(\frac{2}{15}\right)} = 8.8174$ **(for fac. B comparisons)**

[$Note.$ **Tukey (in conjunction with Bonferroni) critical values:**

$\frac{1}{\sqrt{2}}q(.95, \mathbf{3{,}24}) = \frac{\mathbf{3.53}}{\sqrt{2}} = 2.496$ **(for Factor A comparisons)**

$\frac{1}{\sqrt{2}}q(.95, \mathbf{2{,}24}) = \frac{\mathbf{2.92}}{\sqrt{2}} = 2.065$ **(for Factor B comparisons)**

Here one Tukey critical value is larger and one smaller than the Bonferroni cv of 2.391.]

Point estimates:

$\bar{y}_{2..} - \bar{y}_{1..} = 105 - 40 = 65$ **CI:** $\mathbf{65} \pm 10.\mathbf{80}$

$\bar{y}_{2..} - \bar{y}_{3..} = 105 - 65 = 40$ $\mathbf{40} \pm 10.\mathbf{80}$

$\bar{y}_{3..} - \bar{y}_{1..} = 65 - 40 = 25$ $\mathbf{25} \pm 10.\mathbf{80}$

$\bar{y}_{.2.} - \bar{y}_{.1.} = 80 - 60 = 20$ $\mathbf{20} \pm 8.82$

Results **Bonferroni**

Confidence intervals:

$54.20 \le \mu_{2.} - \mu_{1.} \le \mathbf{75.80}$

$29.20 \le \mu_{2.} - \mu_{3.} \le \mathbf{50.80}$

$14.20 \le \mu_{3.} - \mu_{1.} \le 35.80$

$11.18 \le \mu_{.2} - \mu_{.1} \le 28.82$

Part II (30 points.)
Fill in the blanks or circle the most appropriate answer in the problems below.

6. A plot of estimated treatment means that exhibits non-parallel curves for different factor levels indicates the presence of **interaction effects**.

7. If no interaction exists, factor effects are called (a) multiplicative (b) **additive** (c) divisive

8. The study in Part I is an example of a (a) fractional factorial study (b) **complete factorial study** .

9. In a two-factor study, if factor A has main effects, then factor B must have main effects.
True **False**

10. In a two-factor study, the sum of the estimated main effects for each factor is always zero.
True False

11. If the <u>sum</u> of all estimated interactions is zero, then SSAB $=0$.

True **False**

12. If a simple transformation of Y removes most interaction effects and makes them unimportant, then the interactions are called (a) **transformable** (b) nontransformable
.

13. The F test that is recommended to be done first in a two-factor study is the test for
(a) factor A main effects (b) factor B main effects (c) **interaction effects**

14. Factor effects can always be detected by an F test.
True **False**

15. The Kimball inequality provides an (a) **upper bound** (b) lower bound on the family level of significance for the three F tests for interaction and main effects.

16. If $\alpha_1 = \alpha_2 = \alpha_3 = .05$ is the significance level for the three F tests referred to above, the Kimball significance bound is **.142625**, whereas the Bonferroni bound is **.15**.

17. If important nontransformable interactions are detected in a two-factor study, then multiple pairwise comparisons of (a) factor level means **(b) treatment means** is recommended to analyze factor effects.

18. If greater power is desired in detecting differences in a two-factor study, one could
(a) **increase sample sizes** (b) decrease sample sizes .

19. For a two-factor study, SSTR $=$ SSA $+$ SSB $+$ SSAB. **True** False