STAT 6604 Worksheet 1 **Name____KEY________________________**

1. Provide a brief definition or description of the following terms.

• block – a homogeneous group of experimental units.

• completely randomized design – design in which the treatments are allocated to the experimental units completely at random. For a given number of experimental units per treatment, all possible assignments of treatments to experimental units are equally likely.

• confounding – in experimental design, factors or interactions are confounded when the design array is configured so that the effect of one factor is combined with the other. The effect of the individual factors/interactions cannot be isolated by the analysis.

• control group – subjects in the study who do not receive the intervention being studied. The subjects will have similar characteristics as the subjects in the experimental group except for the fact they did not receive the intervention (or they received some standard treatment).

• double blind study – a study where neither the subjects nor the researchers know who specifically is receiving the drug of treatment under study

• experimental unit – the entity to which a specific treatment combination is applied.

• explanatory variable – the explantory variable is one which is used as a predictor of the response variable. Statistical data analysis wants to reveal the effect of the explantory variable on the response variable. In other words, how are responses influenced by the explanatory variable?

• factor – an explanatory variable studied in an investigation that can be set at any one of two or more values.

• interaction – factors interact when the combined effect is not the sum of the individual effects. Interaction occurs when the effect of one factor on a response depends on the level of another factor(s).

• nested (vs. crossed) factors:

– crossed factors: two factors are crossed if every level of one occurs with every level of the other in the experiment.

– nested factors: a factor "A" is nested within another factor "B" if the levels or values of "A" are different for every level or value of "B".

• placebo – a placebo is an inactive treatment or procedure. It literally means 'I do nothing'. The 'placebo effect' (usually a positive or beneficial response) is attributable to the patient's expectation that the treatment will have an effect.

- randomized complete block design – the randomized complete block design is a design in which the subjects are matched according to a variable which the experimenter wishes to control. The subjects are put into groups (blocks) of the same size as the number of treatments. The members of each block are then randomly assigned to different treatment groups.

- replication – An experiment contains replication if at least one treatment is applied independently to two or more experimental units.

- response variable – a characteristic of an experimental unit measured after treatment and analyzed to address the objectives of the experiment

- treatment – a treatment is a specific combination of factor levels whose effect is to be compared with other treatments.

2. The following table contains data describing the degree of wear of samples of synthetic wood veneers from five manufacturers.

Manufacturer					Mean	Stdev
ACME	2.3	2.1	2.4	2.5	2.3250	0.1708
CHAMP	2.2	2.3	2.4	2.6	2.3750	0.1708
AJAX	2.2	2.0	1.9	2.1	2.0500	0.1291
TUFFY	2.4	2.7	2.6	2.7	2.6000	0.1414
XTRA	2.3	2.5	2.3	2.4	2.3750	0.0957

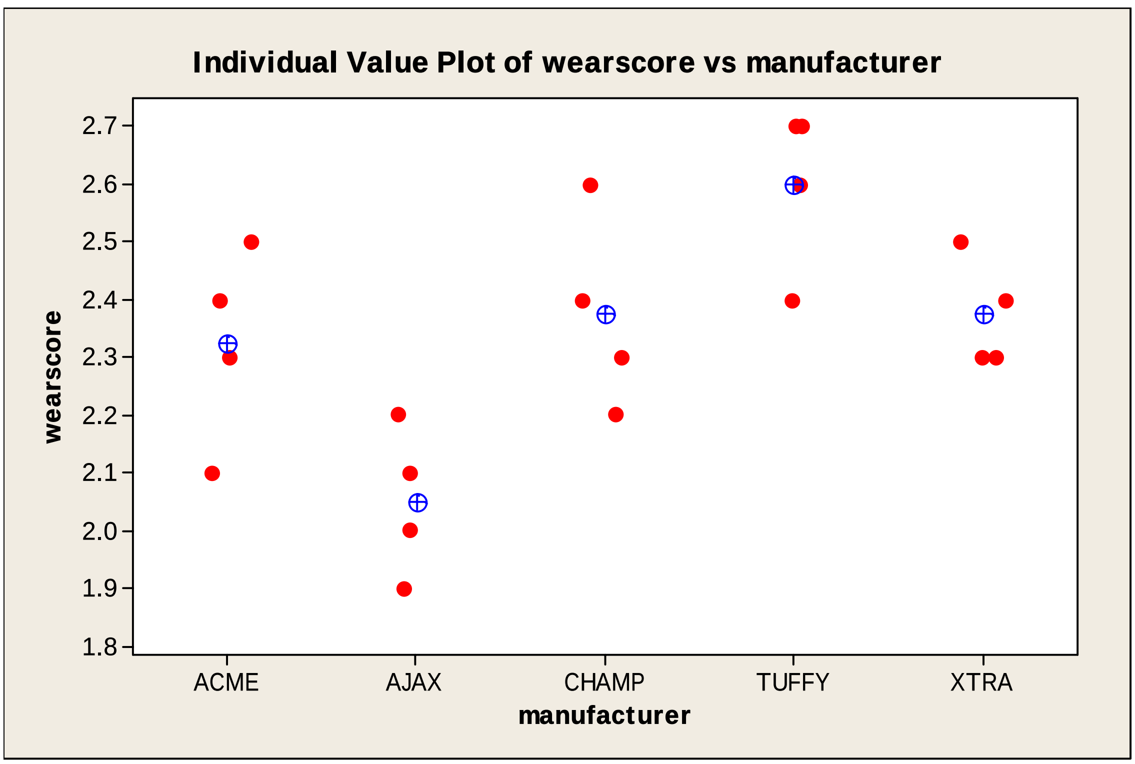
(i) Obtain the sample means and standard deviations to complete the table above.

(ii) Find the observation y_{13} and the residual e_{13}.

$$y_{13} = 2.4$$

$$e_{13} = 2.4 - 2.325 = 0.075$$

(iii) Plot the data (along with treatment sample means) and comment on the plot.



Comment: There appears to be relatively large difference between the Tuffy sample mean and the Champ sample mean. The response variation within each treatment appears fairly constant.

(iii) Complete the ANOVA table for testing the equality of mean wear.

ANOVA TABLE					
Source of Variation	DF	SS	MS	F statistic	p-value
Treatments	4	0.6170	0.1542	7.40	0.002
Error	15	0.3125	0.0208		
Total	19	0.9295			

(iv) Show that the error mean square is a (weighted) average of the sample variances.

$$\text{MSE} = \frac{1}{n_{\text{T}}-r}\sum_{i=1}^{5}(n_i-1)s_i^2$$

$$MSE = 0.0208$$

and

$$\begin{aligned}\frac{1}{n_{\text{T}}-r}\sum_{i=1}^{5}(n_i-1)s_i^2 &= \tfrac{1}{15}(3(0.1708)^2 + 3(0.1291)^2 + 3(0.1708)^2 + 3(0.1414)^2 + 3(0.0957)^2) \\ &= 0.0208\end{aligned}$$

(v) Show that the treatment sum of squares (SSTR) equals $\sum_{i=1}^{5} n_i(\overline{y}_{i.} - \overline{y}_{..})^2$.

$$\begin{aligned}\sum_{i=1}^{5} n_i(\overline{y}_{i.} - \overline{y}_{..})^2 &= 4(2.325-2.345)^2 + 4(2.05-2.345)^2 + 4(2.375-2.345)^2 \\ &+ 4(2.60-2.345)^2 + 4(2.375-2.345)^2 \\ &= 0.617 = SSTR\end{aligned}$$

(vi) Let μ_1 denote the true mean wear score for Acme wood veneer,
μ_2 denote the true mean wear score for Champ wood veneer, etc.

If $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$, then we would expect MSTR to be

$\boxed{\text{(a) relatively small}}$ (b) relatively large

with respect to the MSE.

(vii) If F* denotes the F statistic value, then the following MINITAB command

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MTB> cdf F*;
SUBC> f 4 15.
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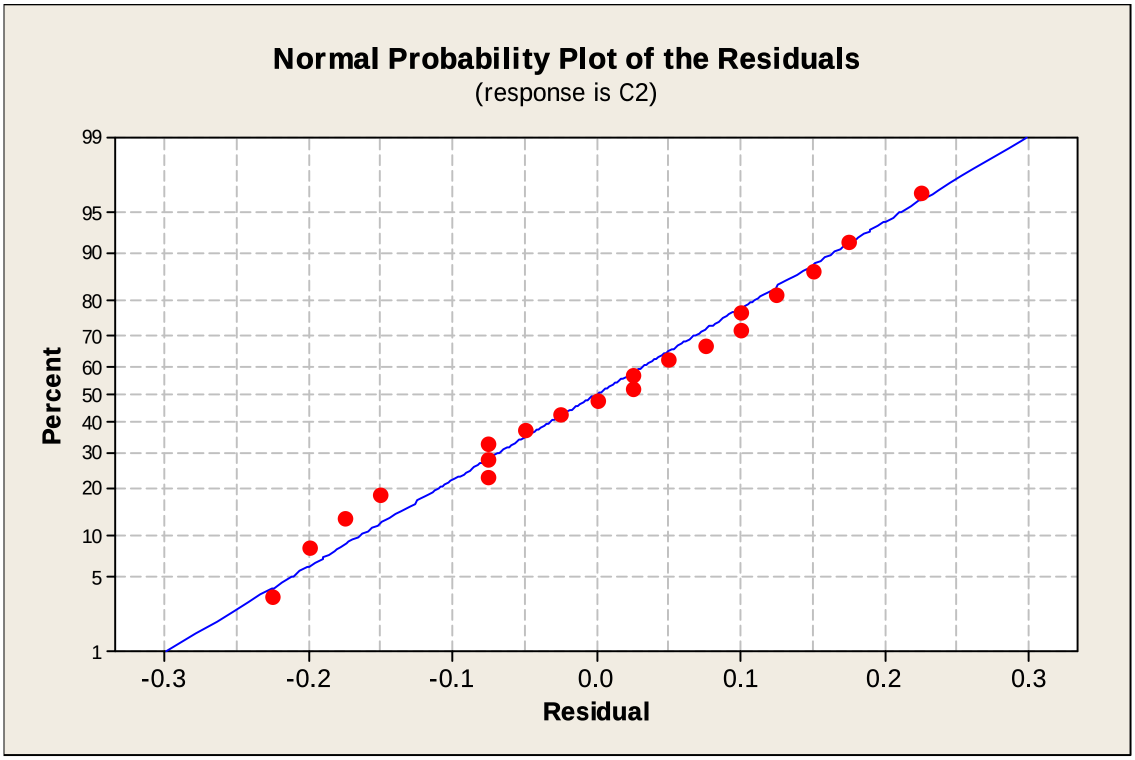
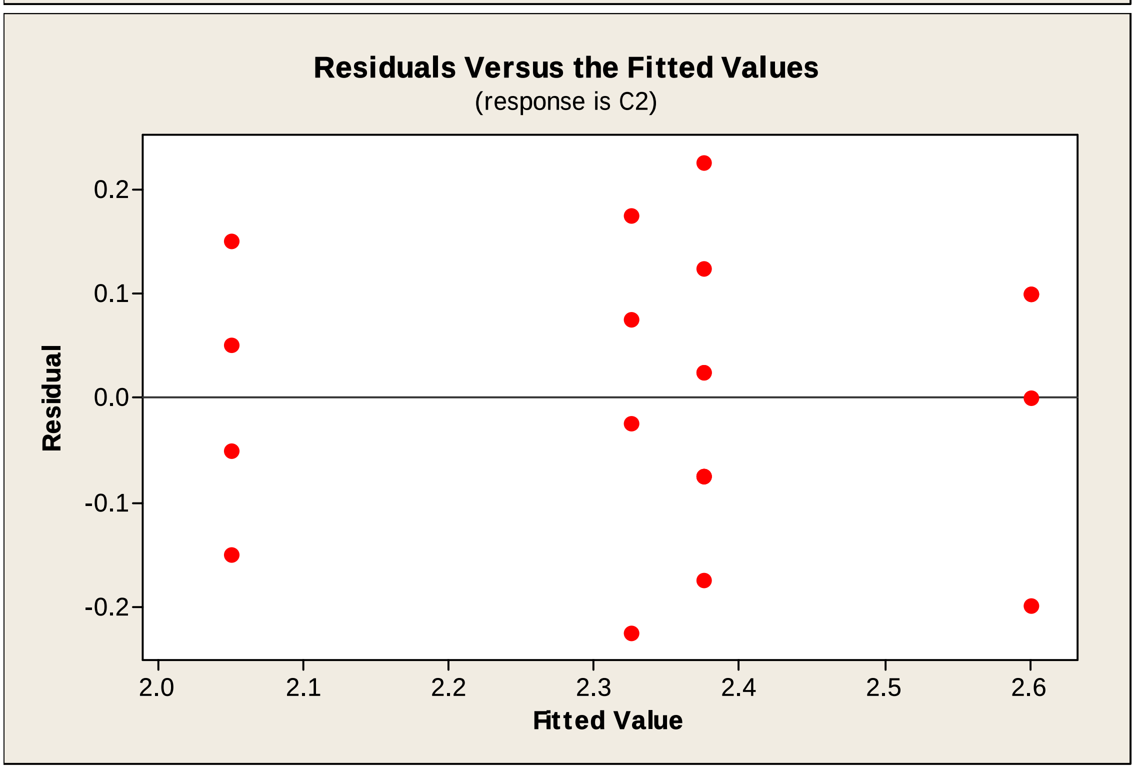
gives us

(a) the p-value of the F test $\boxed{\text{(b) (1 – p-vlaue) of the F test}}$

(viii) The F test is based on the assumptions of normality and homoscedasticity. What do the plots below indicate concerning the validity of those assumptions?





The assumptions appear to be reasonable.

(ix) When testing H_o: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ versus
H_a : not all treatment means are equal,
what is the appropriate conclusion if the level of significance $\alpha = .05$?

Since the p-value is .002, we have sufficient statistical evidence to conclude that not all the mean wear rates are equal.

(x) Use the Tukey method with family confidence level 95% to construct confidence intervals for the 10 comparisons of treatment means.

Using MINITAB ...

Tukey Confidence Intervals with 95% Family Confidence Level

I – J	Confidence Interval for ($\mu_i - \mu_j$)
Tuffy – Ajax	(0.2346, 0.8654)
Tuffy – Acme	(-0.0404, 0.5904)
Tuffy – Xtra	(-0.0904, 0.5404)
Tuffy – Champ	(-0.0904, 0.5404)
Champ – Ajax	(0.0096, 0.6404)
Champ – Acme	(-0.2654, 0.3654)
Champ – Xtra	(-0.3154, 0.3154)
Xtra – Ajax	(0.0096, 0.6404)
Xtra – Acme	(-0.2654, 0.3654)
Acme – Ajax	(-0.0404, 0.5904)

The **Tukey** method is best when all $\binom{r}{2}$ **pairwise comparisons** are done. The method is based on the *studentized range distribution*, which is, for example, the sampling distribution of $\frac{\max(\bar{Y}_i) - \min(\bar{Y}_i)}{\sqrt{\text{MSE}/n}}$ under H_0 and when $n_i = n$. The critical values q for the Tukey method are in Table B.9. We don't require that sample sizes be equal. Also, data snooping in terms of pairwise comparisons is allowed. Confidence intervals have form $\widehat{D} \pm \frac{q}{\sqrt{2}} s\{\widehat{D}\}$ where $\widehat{D}$ denotes the difference of two sample means.

Margin of error for our confidence intervals above:

$$E = \frac{q}{\sqrt{2}} s\{\widehat{D}\} = \frac{q}{\sqrt{2}} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = \frac{4.37}{\sqrt{2}} \sqrt{.0208\left(\frac{1}{4} + \frac{1}{4}\right)}$$

$$= .315$$