REVIEW OF SOME BASIC PROBABILITY

Basic Terminology

- **probability experiment**: any process that has an observable outcome based on chance
- **sample space**: set of all possible outcomes of a probability experiment
- **event**: a subset of the sample space
- **$\overline{E}$, the complement of an event E**: all outcomes in the sample space that are not in event E;
- **impossible event**: an event whose occurrence has probability zero;
  the null set $\emptyset$ is an impossible event
- **certain event:** an event whose occurrence has probability one;
  the sample space S is a certain event
- **mutually exclusive (or disjoint) events**: events that have no common outcomes.
  Events A and B are mutually exclusive if $A \cap B = \emptyset$.
- **independent events**: set of events such that the joint occurrence of any subset equals the product of the respective probabilities. [Below we let juxtaposition denote intersection.]
  Events A and B are independent iff $P(AB) = P(A)\ P(B)$.
  Events A, B, and C are independent iff $P(AB) = P(A)P(B)$,
  $P(AC) = P(A)P(C), P(BC) = P(B)\ P(C)$, and $P(ABC) = P(A)P(B)P(C)$.
- **identical trials**: a sequence of repeated probability experiments performed under identical conditions
- **relative frequency** of event E: the number of trials favorable to E divided by the total number of identical trials in the sequence
- **experiment with equally-likely outcomes**: an experiment in which any outcome is just as likely to occur as any other outcome
- **odds in favor of an event E**: $\frac{P(E)}{P(\overline{E})}$, the probability of E divided by the probability of the complement of E.
- **odds against an event E**: $\frac{P(\overline{E})}{P(E)}$, the probability of the complement of E divided by the probability of E

Three Basic Rules (Kolmogorov Axioms). Let S denote the sample space of an experiment.

1. For any event $A \subseteq S$, $0 \leq P(A) \leq 1$.
2. $P(S) = 1$
3. For a finite or infinite sequence of disjoint events $\{A_i\}$, we have $P(\underset{i}{\cup} A_i) = \sum_i P(A_i)$.

Interpretations of Probability

The basic axioms above form the foundation of probability theory. Probability is sometimes interpreted as (i) long term relative frequency of an event, (ii) the proportion of outcomes in the sample space that are members of the event provided that all outcomes are equally-likely, and (iii) the personal likelihood assigned to an event.

Probability Rules Based on the Basic Axioms

4. $P(\bar{A}) = 1 - P(A)$ where $\bar{A}$ = complement of $A = \{$outcomes in sample space S that are not in $A\}$

5. $P(\emptyset) = 0$ and $P(S) = 1$

6. If $A \subseteq B$, then $P(A) \leq P(B)$

7. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

8. $P(\cup A_i) \leq \sum P(A_i)$

9. Suppose $A_1 \subseteq A_2 \subseteq A_3 \cdots$ and $A = \underset{n}{\cup} A_n$, then $\lim_{n \to \infty} P(A_n) = P(A)$.

10. Suppose $A_1 \supseteq A_2 \supseteq A_3 \cdots$ and $A = \underset{n}{\cap} A_n$, then $\lim_{n \to \infty} P(A_n) = P(A)$.

11. $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i A_j) + \sum_{i<j<k} P(A_i A_j A_k) - \ldots + (-1)^{n+1} P(A_1 A_2 \cdots A_n)$

12. $P(B \mid A) = P(A \cap B)/P(A)$, provided $P(A) > 0$. (Definition of conditional probability)

13. $P(A \cap B) = P(A)\, P(B \mid A) = P(B)\, P(A \mid B)$.

14. $P(\bigcap_{i=1}^{n} A_i) = P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid A_1 A_2) \cdots P(A_n \mid A_1 A_2 A_3 \cdots A_{n-1})$

15. Let $B_1, B_2, \ldots, B_k$ be a partition of S, i.e., a collection of disjoint sets whose union is S.
    Then $P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(B_i)\, P(A \mid B_i)$. (Law of total probability)

16. If $B_1, B_2, \ldots, B_k$ is a partition of S and $P(A) > 0$, then $P(B_1 \mid A) = \dfrac{P(B_1)\, P(A|B_1)}{\sum_{i=1}^{k} P(B_i)\, P(A|B_i)}$.

17. $P(\bigcap_{i=1}^{n} \bar{A}_i) = 1 - P(\bigcup_{i=1}^{n} A_i)$