Economics 6110
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Final Examination
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1. Consider the following model embodying rational expectations:

(IS)	$y_t = -ai_t + \nu_t$
(LM)	$m_t = p_t + b_1 y_t - b_2 i_t + \eta_t$
(AS)	$y_t = c_0(p_t - E_{t-1}p_t) + c_1 y_{t-1}$
(Policy)	$m_t = \gamma i_t + \lambda y_{t-1}$

where all parameters are positive and all time-dependent variables are expressed as natural logarithms except for the nominal interest rate. $E_{t-1}p_t$ is the rational expectation of p_t formed by the private sector at time $t-1$. The stochastic shocks ν_t and η_t are **independent** white noise disturbances with variances σ_ν^2 and σ_η^2, respectively. The coefficient pair (γ, λ) represent the systematic feedback parameters that govern the response of monetary policy to economic events. Take notice that the model ignores all intercept terms.

a. Briefly discuss the **microeconomic** rationale behind the aggregate supply assumption that only unexpected movements in the price level $(p_t - E_{t-1}p_t)$ generate departures of real income y_t from some normal level. (4 points)

b. Determine the rational expectations solution for real income y_t and the price level p_t. (10 points)

c. Find an expression for the the **unconditional** variance of y_t in terms of the variances of the stochastic shocks and the model's exogenous parameters. (4 points)

d. Explain the Sargent-Wallace *policy ineffectiveness proposition* and ascertain whether or not (or in what sense) this proposition holds in the model. Explain the economic rationale behind your result. (4 points)

e. Suppose the only shocks hitting the economy are LM shocks ($\sigma_\nu^2 = 0$). How should the policymaker set the systematic feedback coefficients if it wants to minimize the impact of such shocks on real income y_t? Try to illustrate your answer graphically using the IS-LM/AD-AS apparatus. (6 points)

f. Repeat part (e) assuming that the only shocks hitting the economy are IS shocks ($\sigma_\eta^2 = 0$). (6 points)

2. Consider an economy that is populated by a large number of infinitely-lived identical agents with expected lifetime utility given by

$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$

where c_t and l_t are the date t consumption and leisure allocations, respectively. $\beta \in (0, 1)$ is the exogenous discount factor and E_0 is a mathematical expectations operator conditional on date zero information. The momentary utility function, the time constraint, the aggregate production function, the economy-wide resource constraint, and the law of motion for capital are given by

$$
\begin{aligned}
&u(c_t, l_t) = \ln c_t + \theta \ln l_t \qquad \theta > 0 \\
&l_t + n_t = 1 \\
&f(a_t, k_t, n_t) = a_t k_t^{\alpha} n_t^{1-\alpha} \\
&y_t = f(a_t, k_t, n_t) \\
&y_t = c_t + i_t \\
&k_{t+1} = i_t + (1 - \delta) k_t
\end{aligned}
$$

where y_t, n_t, k_t, and i_t correspond to date t realizations of output, work hours, the capital stock, and investment, respectively. The parameter $\alpha \in (0, 1)$ measures the capital share of output while $\delta \in (0, 1)$ measures the rate of capital depreciation. The exogenous law of motion for the random productivity disturbance a_t is

$$\ln a_t = (1 - \rho) \ln a + \rho \ln a_{t-1} + \varepsilon_t \qquad \rho \in (0, 1) \qquad \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$$

where a corresponds to the unconditional mean of a_t. **Note**: this model does not specify a deterministic growth component so that all relevant variables are already expressed in detrended form.

a. Write down the Lagrangian that corresponds to the social planner's maximization problem defined over the optimal sequence $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$. (5 points)

b. Write down the sequence of equations that fully characterize the optimal plans $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ given the initial values $\{k_0, a_0\}$. Do the optimal allocations derived from the social planner's problem necessarily coincide with the allocations that would prevail in an environment where economic decisions are made in a decentralized manner? Explain briefly. (8 points)

c. Find expressions for the following variables in the steady state as functions of the model's exogenous parameters: $\{r, k/n, y/n, c/n, n\}$. Note: r is the steady state real return to capital investment. (8 points)

d. Discuss how one might calibrate the following parameters: $\{\beta, \delta, \theta\}$. (6 points)

e. Carefully discuss three separate reasons why real output y_t would increase in the aftermath of a positive shock to productivity a_t. (6 points)

3. Consider an economy described by the following set of equations:

$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}$	$\alpha \in (0,1)$	
$\dot{K}(t) = sY(t)$	$s \in (0,1)$	
$\dot{A}(t) = BY(t)^{\phi}$	$B > 0$	$\phi \in (0,1)$
$\dot{L}(t) = nL(t)$	$n > 0$	

The model assumes that all available resources are used to produce a homogeneous final good (Y) according to a standard production function defined over capital (K), labor (L), and knowledge (A). The accumulation of knowledge, on the other hand, occurs as a side effect of goods production. Additions to the capital stock are a constant fraction s of output while labor grows at an exogenous rate n.

a. Find expressions for the growth rates of capital $g_K(t)$ and knowledge $g_A(t)$ in terms of $K(t)$, $A(t)$, $L(t)$, and the parameters of the model. (6 points)

b. Sketch the $\dot{g}_K = 0$ and $\dot{g}_A = 0$ lines in (g_A, g_K) space. Make sure that you label the intercepts. (8 points)

c. Illustrate graphically that this economy converges to a balanced growth path, and find expressions for the growth rates of K, A, and Y on that balanced growth path. Make sure that you provide justification for the stable dynamic behavior of (g_A, g_K). (10 points)

d. Suppose the economy is initially on its balanced growth path. Illustrate graphically how each of the following **permanent** changes affects the position of the economy in (g_A, g_K) space. Demonstrate what happens in the long run as well as what happens at the moment of the change. (9 points)

   (*i*) an increase in n
   (*ii*) an increase in s
   (*iii*) an increase in B