Economics 6110
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Final Examination
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1. Consider the following Keynesian economy where the nominal wage is fixed ($W = \overline{W}$).

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|---|---|---|---|---|---|
| $Y = E(Y, r, G, T)$ | $E_Y \in (0, 1)$ | $E_r < 0$ | $E_G > 0$ | $E_T < 0$ |
| $M/P = L(r + \pi^e, Y, \gamma)$ | $L_i < 0$ | $L_Y > 0$ | $L_\gamma > 0$ | |
| $Y = F(N)$ | $F'(N) > 0$ | $F''(N) < 0$ | | |
| $\overline{W}/P = F'(N)$ | | | | |

Although the nominal wage is fixed while the price level is variable, expected inflation is taken as exogenous in the model and so does not respond to economic shocks. The model variables are defined as follows:

$Y$ = real output
$E$ = planned expenditure
$r$ = real interest rate
$i$ = nominal interest rate
$G$ = government spending (exogenous)
$T$ = government taxes (exogenous)
$M$ = money supply (exogenous)
$P$ = price level
$\pi^e$ = expected inflation (exogenous)
$N$ = employment
$W$ = nominal wage
$\gamma$ = exogenous shock to money demand

a. Calculate the impact on $N$, $r$, and $P$ of a positive shock to liquidity demand ($\gamma$). In other words, find expressions for $dN/d\gamma$, $dr/d\gamma$, and $dP/d\gamma$. Use the restrictions given in the model to sign the derivative expressions. (12 points)

b. Try to illustrate your answer from part (a) graphically. Specifically, show all the adjustments that occur in the market for goods and services (Keynesian cross), the market for real money balances, the IS-LM graph, and the AS-AD graph. Be sure to label all axes and curves carefully and indicate the direction that each curve shifts. Accompany your illustrations with a brief discussion/argument for why the appropriate schedules shift in the manner that you have described. (12 points)

c. Suppose that the cyclical movements of $Y$, $N$, $r$, and $P$ are driven by shocks to liquidity demand. What does the model imply about the cyclical behavior of the **real** wage? Illustrate your answer graphically with a depiction of the labor market for this model. Do your conclusions contradict the evidence about the cyclicality of real wages in U.S. data? (5 points)

d. What does the sticky wage model imply about the behavior of unemployment in the aftermath of a positive liquidity demand shock? Justify your answer. (4 points)

2. Consider an economy that is populated by a large number of infinitely-lived identical agents with expected lifetime utility given by

$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$

where $c_t$ and $l_t$ are the date $t$ consumption and leisure allocations, respectively. $\beta \in (0,1)$ is the exogenous discount factor and $E_0$ is a mathematical expectations operator conditional on date zero information. The momentary utility function, the time constraint, the aggregate production function, the economy-wide resource constraint, and the law of motion for capital are given by

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \theta \frac{l_t^{1-\chi}}{1-\chi} \qquad \sigma > 0 \qquad \chi > 0 \qquad \theta > 0$$
$$l_t + n_t = 1$$
$$f(a_t, k_t, n_t) = a_t k_t^{\alpha} n_t^{1-\alpha}$$
$$y_t = f(a_t, k_t, n_t)$$
$$y_t = c_t + i_t$$
$$k_{t+1} = i_t + (1-\delta) k_t$$

where $y_t$, $n_t$, $k_t$, and $i_t$ correspond to date $t$ realizations of output, work hours, the capital stock, and investment, respectively. The parameter $\alpha \in (0,1)$ measures the capital share of output while $\delta \in (0,1)$ measures the rate of capital depreciation. The exogenous law of motion for the random productivity disturbance $a_t$ is

$$\ln a_t = (1-\rho) \ln a + \rho \ln a_{t-1} + \varepsilon_t \qquad \rho \in (0,1) \qquad \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$$

where $a$ corresponds to the unconditional mean of $a_t$. **Note**: this model does not specify a deterministic growth component so that all relevant variables are already expressed in detrended form.

a. Write down the Lagrangian that corresponds to the social planner's maximization problem defined over the optimal sequence $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$. (5 points)

b. Write down the sequence of equations that fully characterize the optimal plans $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ given the initial values $\{k_0, a_0\}$. Do the optimal allocations derived from the social planner's problem necessarily coincide with the allocations that would prevail in an environment where economic decisions are made in a decentralized manner? Explain briefly. (8 points)

c. Find expressions for the following variables in the steady state as functions of the model's exogenous parameters: $\{r, k/n, y/n, c/n, i/k\}$. Note: $r$ is the steady state real return to capital investment. (8 points)

d. Discuss what is meant by the term *calibration*. How might one calibrate the following parameters: $\{\beta, \delta, \theta\}$? (6 points)

e. Carefully discuss three separate reasons why real output $y_t$ would increase in the aftermath of a positive shock to productivity $a_t$. (6 points)

3. Consider a Research and Development model described by the following set of equations:

$$Y(t) = [(1 - a_K)K(t)]^{\alpha}[A(t)(1 - a_L)L(t)]^{1-\alpha} \qquad \alpha \in (0,1)$$
$$\dot{A}(t) = B[a_K K(t)]^{\theta}[a_L L(t)]^{\gamma} A(t)^{1-\theta} \qquad B > 0 \quad \theta \in (0,1) \quad \gamma \in (0,1)$$
$$\dot{L}(t) = nL(t)$$
$$\dot{K}(t) = sY(t)$$

The model assumes that a fraction of available resources are used to produce a homogeneous final good ($Y$) according to a standard production function defined over capital ($K$), labor ($L$), and knowledge ($A$). The remaining fraction of resources ($a_K$ and $a_L$), on the other hand, are used for the accumulation of new technologies. Additions to the capital stock are a constant fraction $s$ of output. For this model, assume that there is **zero population growth**, that is, $n = 0$. As a result, one can simply normalize the labor input by setting $L(t) = 1$ for all $t$.

a. Find expressions for the growth rates of capital and knowledge, $g_K(t)$ and $g_A(t)$, in terms of $K(t)$, $A(t)$, and the model's underlying parameters. (6 points)

b. Sketch the $\dot{g}_K = 0$ and $\dot{g}_A = 0$ loci in $(g_A, g_K)$ space. (8 points)

c. Prove that the economy converges to a stable balanced growth path. Is that balanced growth path unique? If so, find the growth rates of $K(t)$, $A(t)$, and $Y(t)$ on the balanced growth path. (10 points)

d. Suppose the economy is initially on its balanced growth path. Illustrate graphically how each of the following **permanent** changes affects the position of the economy in $(g_A, g_K)$ space. Demonstrate what happens in the long run as well as what happens at the moment of the change. (10 points)

   (*i*) an increase in $s$
   (*ii*) an increase in $B$
   (*iii*) an increase in $a_k$