Economics 6110
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Final Examination
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1. Consider an economy with an aggregate production function of the Cobb-Douglas variety. The depreciation rate of physical capital (δ), the saving rate (s), and the growth rates of labor (n) and technology (γ) are fixed and exogenous.

$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}$ where $\alpha \in (0,1)$
$\dot{L}(t) = nL(t)$
$\dot{A}(t) = \gamma A(t)$
$Y(t) = C(t) + I(t) + G(t)$
$G(t) = \tau Y(t)$ where $\tau \in (0,1)$
$I(t) = s(Y(t) - G(t))$
$\dot{K}(t) = I(t) - \delta K(t)$

The continuous time variables are defined as: $Y(t)$ - output, $K(t)$ - capital, $L(t)$ - labor, $A(t)$ - technology, $C(t)$ - consumption, $I(t)$ - investment, $G(t)$ - government spending. Furthermore, define any variable $\dot{X}(t) = \frac{dX(t)}{dt}$. Notice that the only difference between this economy and the standard Solow growth economy is the inclusion of government spending, which is assumed to be a constant fraction τ of output every period.

a. Explain what is meant by a *rival* versus a *non-rival* good. Which inputs in the production process are rival and which are non-rival? (3 points)

b. Show that the key differential equation describing the temporal behavior of $k(t)$, the capital stock per *effective* worker, is given by

$\dot{k}(t) = s(1-\tau)k(t)^{\alpha} - (\delta + n + \gamma)k(t)$ where $k(t) \equiv \frac{K(t)}{A(t)L(t)}$. (7 points)

c. Find expressions for k^*, y^*, and g^* (the steady state values of capital, output, and government spending per effective worker) as functions of s, n, γ, α, τ, and δ. (7 points)

d. Examine the stability of the system and characterize the adjustment of the capital stock towards its steady state. Be sure to illustrate your answer with the appropriate graph showing the adjustment process. (4 points)

e. Suppose the economy experiences a permanent rise in government spending (modeled as an increase in τ). Illustrate graphically what happens to the steady state value of output per effective worker. In addition, explain what happens to the **growth rate** of *per capita* output in the immediate aftermath of the rise in government spending as well as during the transition to the new steady state equilibrium. Try to justify your answer with the appropriate mathematical and graphical arguments. (5 points)

f. Find an expression for the elasticity of $y*$ with respect to τ in terms of the model parameters. (4 points)

g. Find an expression for the *golden rule* level of capital per effective worker k^g in terms of the model parameters. What saving rate s^g is needed to reach the golden rule steady state. Identify the location of k^g on the appropriate graph. (5 points)

2. Consider the following version of the Keynesian IS-LM model:

$$Y = C(Y-T) + I(r) + G \qquad C'(Y-T) > 0 \quad I'(r) < 0$$
$$M/P = L(r+\pi^e, Y-T) \qquad L_i < 0 \qquad L_{Y-T} > 0$$

In this economy, the demand for real money balances depends on the level of *disposable* income $Y-T$ rather than on the level of actual income. The model variables are defined as follows:

Y = output
C = consumption
I = investment
G = government spending
T = taxes
r = real interest rate
i = nominal interest rate
M = money supply
P = price level
π^e = expected inflation

a. Calculate the impact on Y and r of a marginal increase in taxes T. In other words, find expressions for dY/dT and dr/dT. Use the restrictions given in the model to sign the derivative expressions. Are the effects positive, negative, or ambiguous? (15 points)

b. Try to illustrate your answer from part a. graphically. Specifically, show all the adjustments that occur in the market for goods and services (Keynesian cross), the market for real money balances, and the IS-LM graph. Be sure to label all axes and curves carefully and indicate the direction that each curve shifts. Accompany your illustrations with a brief discussion/argument for why the appropriate schedules shift in the manner that you have described. (10 points)

3. Consider the following version of the Ramsey neoclassical growth model:

$U = \sum_{t=0}^{\infty} \beta^t \left(\frac{-\exp(-\theta c_t)}{\theta} \right) \qquad \theta > 0,\ \beta = \frac{1}{1+\rho} < 1$

$\sum_{t=0}^{\infty} \frac{c_t}{\overline{r}_{0,t}} = k_0 + \sum_{t=0}^{\infty} \frac{w_t}{\overline{r}_{0,t}} \qquad$ where $\overline{r}_{0,t} = (1+r_0)(1+r_1)\dots(1+r_t)$, k_0 is given

$\lim_{t\to\infty} \frac{k_{t+1}}{\overline{r}_{0,t}} = 0 \qquad$ (No Ponzi Condition)

Households in this economy maximize lifetime utility U by choosing an optimal consumption plan $\{c_t\}_{t=0}^{\infty}$ subject to their lifetime budget constraint (given by the second equation) and the no-ponzi-game-condition. Households take factor prices w_t and r_t as given when formulating optimal consumption/saving plans. In this model, the growth rates of technology g and the population n are both equal to zero. Thus, one need not draw any distinction between economy-wide variables and their *intensive* form (per effective worker) counterparts.

a. Formally construct the lagrangian that characterizes the households optimization problem. Be sure to indicate what variables are being maximized over. (3 points)

b. Derive the full set of first-order-conditions that characterize the solution to the household's optimization problem. (5 points)

c. Derive the intertemporal Euler condition relating the change in consumption to the real interest rate r_t and the discount rate ρ. (7 points)

d. Interpret the Euler equation derived in part c. Provide an intuitive explanation for why this condition must hold along an optimal consumption trajectory. (4 points)

e. Assume that firms transform capital and labor into output using the following neoclassical production function: $Y = K^{\alpha}L^{1-\alpha}$. Furthermore, assume that capital depreciates at a constant rate δ. Find expressions for the steady state values of capital k^* and consumption c^* in terms of ρ, δ, and α. (7 points)

f. Construct a phase diagram in (k_t, c_t) space that illustrates the joint dynamics of the capital stock and consumption over time. Indicate what regions of the phase diagram lead to stable dynamics and what regions lead to unstable dynamics. Justify your answer. Does this model exhibit the *saddle path* property? (7 points)

g. Suppose there is an unanticipated, permanent **increase** in the discount rate ρ. Illustrate graphically how this affects the phase diagram constructed in part f. What happens to the steady state values k^* and c^* following the increase in ρ? Assuming the the economy is initially on its balanced growth path before the increase in ρ, describe the ensuing transition path to the new steady state. (7 points)