Economics 6110
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Final Examination
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1. Consider the following Keynesian model embodying rational expectations:

(IS)	$y_t = -a[i_t - E_{t-1}(p_{t+1} - p_t)]$
(LM)	$m_t - p_t = b_1 y_t - b_2 i_t + \eta_t$
(AS)	$y_t = c(p_t - E_{t-1}p_t) + u_t$
(Policy)	$m_t = \gamma\eta_t + \lambda u_t$

where $\{a, b_1, b_2, c\}$ are positive and all time-dependent variables are expressed as natural logarithms except for the nominal interest rate. E_{t-1} is a mathematical expectations operator conditional on information available through date $t-1$. The stochastic shocks η_t and u_t are **independent** white noise disturbances with variances σ_η^2 and σ_u^2, respectively. The coefficient pair (γ, λ) represent the systematic feedback parameters that govern the response of monetary policy to economic shocks.

a. Determine the rational expectations solution for real output y_t and the price level p_t. (8 points)

b. Find expressions for the **unconditional** variances of y_t and p_t in terms of the variances of the stochastic shocks and the model's exogenous parameters. (6 points)

c. Suppose that the only shocks hitting the economy are LM shocks ($\sigma_u^2 = 0$). What value of γ would minimize the impact of LM shocks on real output y_t? What value would minimize the impact on the price level? Is there any tension between these two policy goals? (7 points)

d. Suppose that the only shocks hitting the economy are supply shocks ($\sigma_\eta^2 = 0$). What value of λ would minimize the impact of supply shocks on real output y_t? What value would minimize the impact on the price level? Is there any tension between these two policy goals? (7 points)

e. Discuss the Sargent-Wallace *policy ineffectiveness proposition* and explain why it does not hold in this model. (5 points)

2. Consider the following version of the Solow growth model:

$$Y(t) = [uK(t)]^\alpha [A(t)L(t)]^{1-\alpha} \qquad \alpha \in (0,1),\ u \in [0,1]$$
$$\dot{L}(t) = nL(t)$$
$$\dot{A}(t) = gA(t)$$
$$\dot{K}(t) = sY(t) - \delta K(t)$$

where the depreciation rate of physical capital (δ), the saving rate (s), as well as the growth rates of labor (n) and technology (g) are constant and exogenous. The parameter u affects production by altering the intensity with which capital is utilized. Capital is fully utilized when $u = 1$. All capital is left idle when $u = 0$.

a. Derive the intensive form production function for this model. (4 points)

b. Show that the key differential equation describing the temporal behavior of $k(t)$, the capital stock per *effective* worker, is given by

$$\dot{k}(t) = sk(t)^\alpha u^\alpha - (\delta + n + g)k(t) \qquad \text{where } k(t) \equiv \frac{K(t)}{A(t)L(t)}.$$ (8 points)

c. Find expressions for the steady-state values of capital (k^*) and output (y^*) per effective unit of labor as functions of the parameters of the model. (6 points)

d. Suppose that there is an exogenous *increase* in the capital utilization rate. Illustrate graphically what happens to k^* and y^*. (5 points)

e. Describe the path of the **growth rate** of per capita output (Y/L) at the moment of the increase in u and during the transition to the new balanced growth path. Illustrate your answer with the appropriate graphs. (5 points)

f. Find an expression for the *golden rule* level of capital per effective worker k^{gr} in terms of the model parameters. What saving rate is needed to reach the golden rule steady state? Identify the location of k^{gr} on the appropriate graph. (5 points)

3. Consider a Diamond overlapping generations economy where individuals live for two periods and have the following lifetime utility function:

$$U_t = \ln C_{1,t} + \frac{1}{1+\rho} \ln C_{2,t+1} \qquad \rho > 0$$

$C_{1,t}$ refers to the time t consumption of the young generation born in period t, and $C_{2,t}$ is the time t consumption of the old generation born in period $t-1$. Firms in this economy combine labor (L_t) supplied *inelastically* by the young generation with capital (K_t) accumulated by the old generation to produce output according to the neoclassical production function $Y_t = F[K_t, L_t]$. Young people supply 1 unit of labor each and divide labor income between consumption in period t and saving. In period $t+1$, the ensuing old cohort consumes the saving plus interest. The old generation does not work. Labor supply grows at a constant rate n per period.

In addition to consumption and private saving, the government coordinates a *pay-as-you-go* social security system that provides supplemental income for the old generation. Specifically, the government collects lump-sum taxes T_t from each young individual in period t and redistributes the proceeds equally to all old individuals in period t.

a. Construct the budget constraint that young individuals will face in period t as well as the constraint that will be faced in period $t+1$ when they are old. (5 points)

b. Given values of the real wage w_t, the real interest rate r_{t+1}, and the social security taxes/transfers that occur over an individual's lifetime, derive the optimal private savings function s_t for an individual born at date t. (6 points)

c. Derive analytic expressions for $\frac{\partial s_t}{\partial w_t}$ and $\frac{\partial s_t}{\partial r_{t+1}}$. These derivatives determine the partial equilibrium effects on private saving from an increase in the real wage and the interest rate. (5 points)

d. Consider the case in which social security taxes are constant over time, that is, $T_t = T_{t+1} = T$. Determine the partial equilibrium effect of an increase in social security taxes on private saving. Under what conditions does a unit increase in social security taxes change private saving by **more than** one unit? Explain. (6 points)

e. Derive the general equilibrium equation describing the evolution of capital per worker ($k_t = K_t/L_t$) over time. This equation implies a relationship between k_{t+1} and k_t. (5 points)

f. Assume that this economy converges to a steady state that is **unique** and **stable**. Prove that a unit increase in steady state social security taxes/transfers will decrease the steady state level of capital per worker. That is, prove that unfunded social security programs tend to decrease capital per worker in the long run. (7 points)