Macroeconomics Qualifying Exam
Part I (Economics 6110)
22 August 2008

1. Suppose the economy is described by the following set of equations:

$$\begin{array}{ll} Y = C(Y) + I(r) + G & C_Y \in (0,1),\ I_r < 0 \\ M/P = L(Y, r) & L_Y > 0,\ L_r < 0 \\ Y = F(\bar{K}, E) & F_E > 0,\ F_{EE} < 0 \\ F_E = q/P & \end{array}$$

where E measures the quantity of energy (oil) input into the production process and q is the exogenous nominal price of energy per unit. All other variables have their usual interpretation. The first two equations are standard Keynesian IS and LM curves. The third equation expresses output as a function of a fixed amount of capital (labor is supplied inelastically) and a variable quantity of energy (oil) input. The fourth equation equates the marginal product of energy with its real unit cost.

a. Compute and sign the effects of a unit rise in q on output Y and the price level P.

b. Try to illustrate your answer in part a with the appropriate graphs.

c. Suppose that investment is completely inelastic ($I_r = 0$). How does this assumption alter your findings from part a? Illustrate graphically where helpful.

2. Consider the following model with physical and human capital:

$$
\begin{aligned}
&Y(t) = [(1-a_K)K(t)]^{\alpha}[(1-a_H)H(t)]^{1-\alpha} \qquad \alpha \in (0,1) \quad a_K \in (0,1) \quad a_H \in (0,1)\\
&\dot{K}(t) = sY(t) - \delta_K K(t)\\
&\dot{H}(t) = B[a_K K(t)]^{\gamma}[a_H H(t)]^{\phi}[A(t)L(t)]^{1-\gamma-\phi} - \delta_H H(t) \qquad \gamma > 0 \quad \phi > 0 \quad \gamma + \phi < 1\\
&\dot{L}(t) = nL(t)\\
&\dot{A}(t) = gA(t)
\end{aligned}
$$

where a_K and a_H are the fractions of the stocks of physical and human capital used in the education sector and δ_K and δ_H are their corresponding rates of depreciation.

This model assumes that human capital is produced in its own sector with its own production function. Bodies (L) are useful only as something to be educated, not as an input into the production of final goods. Similarly, knowledge (A) is useful only as something that can be conveyed to students, not as a direct input to goods production.

a. Define $k \equiv K/(AL)$ and $h \equiv H/(AL)$. Derive the differential equations describing the paths for physical and human capital per effective worker. That is, derive equations for $\dot{k}(t)$ and $\dot{h}(t)$.

b. Find an equation describing the set of combinations of h and k such that $\dot{k} = 0$. Sketch this locus of points in (h, k) space. Do the same for $\dot{h} = 0$.

c. Does this economy have a balanced growth path? If so, is it unique? If so, is it stable? Justify your answer with the appropriate arguments. Find expressions for the steady state values of k and h as functions of the model's underlying parameters. What are the growth rates of output per person, physical capital per person, and human capital per person on the balanced growth path?

d. Suppose the economy is initially on a balanced growth path, and that there is a permanent increase in s. How does this change affect the path of output per person over time? Support your answer with the appropriate graphical argument.