

Macroeconomics Qualifying Exam
Part I (Economics 6110)
7 June 2006

1. Consider an economy with an aggregate production function of the Cobb-Douglas variety. The depreciation rate of physical capital (δ), the saving rate (s), and the growth rates of labor (n) and technology (g) are constant and exogenous.

$$Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha} \quad \text{where } \alpha \in (0, 1)$$
$$\dot{L}(t) = nL(t)$$
$$\dot{A}(t) = gA(t)$$

The continuous time variables are defined as: $Y(t)$ - output, $K(t)$ - physical capital, $L(t)$ - labor, $A(t)$ - knowledge or technology. Furthermore, define any variable $\dot{X}(t) \equiv \frac{dX(t)}{dt}$.

- a. Explain what is meant by a *rival* versus a *non-rival* good. Which inputs in the production process are rival and which are non-rival?
- b. A constant fraction s of the economy's output is saved and directed towards capital accumulation. Thus, the aggregate capital stock evolves according to the following: $\dot{K}(t) = sY(t) - \delta K(t)$. Show that the differential equation describing the behavior of $k(t)$, the capital stock per *effective* worker, is given by

$$\dot{k}(t) = sk(t)^\alpha - (\delta + n + g)k(t)$$

$$\text{where } k(t) \equiv \frac{K(t)}{A(t)L(t)}.$$

- c. Find expressions for k^* and y^* (the steady-state levels of capital and output per effective worker) as functions of s , n , δ , g , and α .
- d. Examine the stability of the system and characterize the adjustment of the capital stock towards its steady state. Be sure to illustrate your answer with the appropriate graph showing the adjustment process.
- e. Suppose that the economy experiences a permanent rise in the saving rate from s_1 to s_2 . Illustrate graphically what happens to the steady-state level of output per effective worker. In addition, explain what happens to the **growth rate** of *per capita* output ($\frac{Y}{L}$) once the economy reaches a new balanced growth path. Does the increase in saving have any effect on the long-run growth rate of per capita output? Justify your answer.
- f. Find an expression for the elasticity of y^* with respect to s in terms of the model parameters. Assuming that $\alpha = \frac{1}{3}$, how much does a rise in the saving rate from $s_1 = .10$ to $s_2 = .11$ change y^* ?
- g. Explain what happens to the **growth rate** of *per capita* output in the immediate

aftermath of the rise in the saving rate as well as during the *transition* to the new steady state equilibrium. Try to justify your answer with the appropriate mathematical and graphical arguments.

2. Consider the following model embodying rational expectations:

$$\begin{aligned} \text{(IS)} \quad & y_t = a_1 y_{t-1} - a_2 [i_t - E_{t-1}(p_{t+1} - p_t)] \\ \text{(LM)} \quad & m_t = p_t + b_1 y_t - b_2 i_t + \eta_t \\ \text{(AS)} \quad & y_t = c_1 (p_t - E_{t-1} p_t) + u_t \\ \text{(Policy)} \quad & m_t = \gamma u_t + \lambda y_{t-1} \end{aligned}$$

where $\{a_1, a_2, b_1, b_2, c_1\}$ are positive and all time-dependent variables are expressed as natural logarithms except for the nominal interest rate. E_{t-1} is a mathematical expectations operator conditional on information available through date $t - 1$. The stochastic shocks η_t and u_t are **independent** white noise disturbances with variances σ_η^2 and σ_u^2 , respectively. The coefficient pair (γ, λ) represent the systematic feedback parameters that govern the response of monetary policy to economic events.

- a. Briefly discuss a **microeconomic** rationale behind the aggregate supply assumption that only unexpected movements in the price level $(p_t - E_{t-1} p_t)$ generate departures of real output y_t from some normal level.
- b. Determine the rational expectations solution for real output y_t and the price level p_t .
- c. Find an expression for the **unconditional** variance of y_t in terms of the variances of the stochastic shocks and the model's exogenous parameters.
- d. Explain the Sargent-Wallace *policy ineffectiveness proposition* and ascertain whether or not (or in what sense) this proposition holds in the model. Explain the economic rationale behind your result.
- e. Suppose that the only shocks hitting the economy are supply shocks ($\sigma_\eta^2 = 0$). How should the policymaker set the systematic feedback coefficients if it wants to minimize the impact of supply shocks on real output y_t ?
- f. Repeat part (e) assuming that the goal of monetary policy is to minimize the impact of supply shocks on the price level p_t ?