Macroeconomics Qualifying Exam
Part I (Economics 6110)
6 June 2007

1. Consider an economy described by the following set of equations:

$$\begin{aligned} Y(t) &= K(t)^{\alpha}[A(t)L(t)]^{1-\alpha} && \alpha \in (0,1) \\ A(t) &= BK(t)^{\phi} && B > 0,\ \phi > 0 \\ \dot{K}(t) &= sY(t) && s \in (0,1) \\ \dot{L}(t) &= nL(t) && n \in [0,1) \end{aligned}$$

The model assumes that all available resources are used to produce a homogeneous final good (Y) according to a standard production function defined over capital (K), labor (L), and knowledge (A). The accumulation of knowledge occurs merely as a side effect of the production of new capital, known as *learning-by-doing*, and not as the result of directing certain resources to a "research and development" sector. Additions to the capital stock are a constant fraction s of output while labor grows at an exogenous rate n. Denote any variable $\dot{X}(t) \equiv \frac{dX(t)}{dt}$.

a. Find expressions for the growth rates of capital and knowledge, $g_K(t)$ and $g_A(t)$, in terms of $K(t)$, $L(t)$, and the model's underlying parameters.

b. Assuming that $\phi < 1$ and $n > 0$, illustrate graphically using a *phase diagram* that the economy converges to a balanced growth path. Find expressions for the growth rates of K, A, and Y on that balanced growth path. Be sure to justify your answer with appropriate explanations where needed.

c. Explain why this model is an example of **endogenous** growth, whereas the Solow model is an example of **exogenous** growth. How does the population growth rate n impact the long-run growth rate of per capita income in the present model? How is this different from the Solow model?

d. Assume that $\phi = 1$ and $n = 0$. Does the economy converge to a stable balanced growth path in this case? If so, find an expression for the long-run growth rate of Y. If not, explain why a stable path does not exist. How does an increase in the saving rate s affect the long-run growth rate of output in this case?

2. Consider the following version of the neoclassical growth model discussed in class:

$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$ $\theta > 0,\ \beta \in (0,1)$

$\sum_{t=0}^{\infty} \frac{c_t}{\bar{r}_{0,t}} \leq k_0 + \sum_{t=0}^{\infty} \frac{w_t}{\bar{r}_{0,t}}$ $\bar{r}_{0,t} = \prod_{j=0}^{t}(1+r_j),\ k_0$ given

$\lim_{t\to\infty} \frac{k_{t+1}}{\bar{r}_{0,t}} = 0$ (No Ponzi Condition)

Households in this economy maximize lifetime utility U by choosing an optimal consumption plan $\{c_t\}_{t=0}^{\infty}$ subject to the lifetime budget constraint (second equation) and the no-ponzi-game-condition. Households take the price of labor w_t and the price of capital r_t as given when formulating optimal consumption/saving plans. Additionally, the growth rates of technology and the population are both equal to zero. Thus, one need not draw any distinction between economy-wide variables and their intensive form (per effective worker) counterparts.

a. Provide an interpretation of the lifetime budget constraint. What is a ponzi scheme? How does the no-ponzi-game-condition rule out such schemes?

b. Construct the lagrangian that characterizes the households optimization problem. Indicate what variables are being maximized over.

c. Derive the full set of first-order-conditions that characterize the solution to the household's maximization problem.

d. Derive the intertemporal Euler condition relating the change in consumption to the real interest rate r_t and the discount factor β.

e. Interpret the Euler condition derived in part d. Explain intuitively why this condition must hold along an optimal consumption trajectory.

f. Assume that firms transform capital and labor into output using the production function $Y = K^{\alpha}L^{1-\alpha}$ and that capital depreciates at a constant rate δ. Find expressions for the steady state values of capital k^* and consumption c^* in terms of β, δ, and α.

g. Construct a *phase diagram* in $(k_t,\ c_t)$ space that illustrates the joint dynamics of capital and consumption over time. Indicate what regions of the phase diagram lead to stable dynamics and what regions lead to unstable dynamics. Does this model exhibit the *saddle path* property? Justify your answer fully.

h. Suppose there is an unanticipated permanent **reduction** in the discount factor β. Illustrate graphically how this affects the phase diagram constructed in part g. Illustrate graphically what happens to the steady state values k^* and c^* following the reduction in β. Assuming that the economy is initially on its balanced growth path before the fall in β, describe the ensuing transition path to the new steady state.