Macroeconomics Qualifying Exam
Part I (Economics 6110)
4 June 2008

1. Consider an economy with an aggregate production function of the Cobb-Douglas variety. The depreciation rate of physical capital (δ), the saving rate (s), and the growth rates of labor (n) and technology (g) are constant and exogenous.

$$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha} \qquad \text{where } \alpha \in (0,1)$$
$$\dot{L}(t) = nL(t)$$
$$\dot{A}(t) = gA(t)$$

The continuous time variables are defined as: $Y(t)$ - output, $K(t)$ - physical capital, $L(t)$ - labor, $A(t)$ - knowledge or technology. Furthermore, define any variable $\dot{X}(t) \equiv \frac{dX(t)}{dt}$.

a. Explain what is meant by a *rival* versus a *non-rival* good. Which inputs in the production process are rival and which are non-rival?

b. A constant fraction s of the economy's output is saved and directed towards capital accumulation. Thus, the aggregate capital stock evolves according to the following: $\dot{K}(t) = sY(t) - \delta K(t)$. Show that the differential equation describing the behavior of $k(t)$, the capital stock per *effective* worker, is given by

$$\dot{k}(t) = sk(t)^{\alpha} - (\delta + n + g)k(t)$$

where $k(t) \equiv \frac{K(t)}{A(t)L(t)}$.

c. Find expressions for k^* and y^* (the steady-state levels of capital and output per effective worker) as functions of s, n, δ, g, and α.

d. Examine the stability of the system and characterize the adjustment of the capital stock towards its steady state. Be sure to illustrate your answer with the appropriate graph showing the adjustment process.

e. Suppose that the economy experiences a permanent rise in the saving rate from s_1 to s_2. Illustrate graphically what happens to the steady-state level of output per effective worker. In addition, explain what happens to the **growth rate** of *per capita* output ($\frac{Y}{L}$) once the economy reaches a new balanced growth path. Does the increase in saving have any effect on the long-run growth rate of per capita output? Justify your answer.

f. Explain what happens to the **growth rate** of *per capita* output in the immediate aftermath of the rise in the saving rate as well as during the *transition* to the new steady state equilibrium. Try to justify your answer with the appropriate mathematical and graphical arguments.

g. Find an expression for the *golden rule* level of capital per effective worker k^g in terms of the model parameters. What saving rate s^g is needed to reach the golden rule steady state. Identify the location of k^g on the appropriate graph.

2. Consider an economy consisting of a constant population of infinitely lived individuals. The representative individual maximizes the expected value of lifetime utility given by

$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(c_t, \nu_t),$

where $\rho > 0$ and E_0 is an expectations operator conditional on date zero information. The momentary utility function is given by $U(c_t) = c_t - \theta(c_t^2 + \nu_t)$, where $\theta > 0$. Assume that c_t is always in the range where $U'(c) > 0$. The stochastic preference shock $\nu_t \sim iid(0, \sigma_\nu^2)$.

In this economy, output is linear in capital, plus an additive technology disturbance: $y_t = ak_t + e_t$. There is no depreciation; thus $k_{t+1} = k_t + i_t$, where investment i_t and consumption c_t are related to output by $y_t = c_t + i_t$. Assume that the production coefficient $a = \rho$. Finally, the technology shock follows a first-order autoregressive process: $e_t = \phi e_{t-1} + \varepsilon_t$, where $-1 < \phi < 1$ and $\varepsilon_t \sim iid(0, \sigma^2)$.

a. Write down the Lagrangian that corresponds to the social planner's maximization problem defined over optimal sequences $\{c_t, k_{t+1}\}_{t=0}^{\infty}$.

b. Derive the set of first order necessary conditions as well as the transversality condition that, together, fully characterize the optimal plan $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ given the initial values $\{k_0, e_0, \nu_o\}$. Are the allocations consistent with this problem equivalent to the ones that would prevail in a decentralized, competitive environment? Explain briefly.

c. Find the consumption Euler equation relating c_t to expectations of c_{t+1}.

d. Guess that the time invariant policy function for consumption takes the form $c_t = \alpha + \beta k_t + \gamma e_t + \eta \nu_t$, where $\beta \neq 0$. Given this guess, what is k_{t+1} as a function of k_t, e_t, and ν_t?

e. What values must the parameters α, β, γ, and η take for the consumption Euler equation in part c to be satisfied for all values of k_t, e_t, and ν_t? In other words, find the unique time invariant policy functions $k_{t+1} = k(k_t, e_t, \nu_t)$ and $c_t = c(k_t, e_t, \nu_t)$ that solve the real business cycle model.

f. Describe the effects of a one-time shock to ν_t of the size $(1 + \rho)$ on the paths of c_t, k_t, and y_t.

g. Assume that $\phi = 0$. Describe the effects of a one-time shock to e_t of the size $(1 + \rho)$ on the paths of c_t, k_t, and y_t.