

1. Consider a Research and Development model described by the following set of equations:

$$\begin{aligned}
 Y(t) &= [(1 - a_K)K(t)]^\alpha [A(t)(1 - a_L)L(t)]^{1-\alpha} & \alpha \in (0, 1) \\
 \dot{A}(t) &= B[a_K K(t)]^\theta [a_L L(t)]^\gamma A(t)^{1-\theta} & B > 0 \quad \theta \in (0, 1) \quad \gamma \in (0, 1) \\
 \dot{L}(t) &= nL(t) \\
 \dot{K}(t) &= sY(t)
 \end{aligned}$$

The model assumes that a fraction of available resources are used to produce a homogeneous final good (Y) according to a standard production function defined over capital (K), labor (L), and knowledge (A). The remaining fraction of resources (a_K and a_L), on the other hand, are used for the accumulation of new technologies. Additions to the capital stock are a constant fraction s of output. For this model, assume that there is **zero population growth**, that is, $n = 0$. As a result, one can simply normalize the labor input by setting $L(t) = 1$ for all t .

- a. Find expressions for the growth rates of capital and knowledge, $g_K(t)$ and $g_A(t)$, in terms of $K(t)$, $A(t)$, and the model's underlying parameters.
- b. Sketch the $\dot{g}_K = 0$ and $\dot{g}_A = 0$ loci in (g_A, g_K) space.
- c. Prove that the economy converges to a stable balanced growth path. Is that balanced growth path unique? If so, find the growth rates of $K(t)$, $A(t)$, and $Y(t)$ on the balanced growth path.
- d. Suppose the economy is initially on its balanced growth path. Illustrate graphically how each of the following **permanent** changes affects the position of the economy in (g_A, g_K) space. Demonstrate what happens in the long run as well as what happens at the moment of the change.
 - (i) an increase in s
 - (ii) an increase in B
 - (iii) an increase in a_k

2. Consider the following economy where the price level is completely fixed ($P = \bar{P} = 1$).

$$\begin{array}{ll} Y = C(Y, \gamma) + I(r) + G & C_Y \in (0, 1), C_\gamma > 0, I_r < 0 \\ M = L(r, Y) & L_r < 0, L_Y > 0 \\ Y = F(N) & F'(N) > 0, F''(N) < 0 \\ N = t(W) & t'(W) > 0 \end{array}$$

Because the price level is fixed, expected inflation is equal to zero at all times and the nominal interest is equal to the real interest rate. The model variables are defined as follows:

Y = real output

r = interest rate

G = government spending (exogenous)

M = money supply (exogenous)

N = employment

W = real wage (also the money wage since $P = 1$)

γ = exogenous shock to consumption demand

- a. Calculate the impact on N , Y , and W of a positive shock to consumption demand (γ). In other words, find expressions for $dN/d\gamma$, $dY/d\gamma$, and $dW/d\gamma$. Can you sign the expressions?
- b. Try to illustrate your answer from part (a) graphically. Specifically, show the adjustments that occur in the market for goods and services (Keynesian cross), the market for real money balances, the labor market, the IS-LM graph, and also the AS-AD graph. Please label all axes and curves carefully.
- c. Suppose that cyclical movements in Y , N , and W are driven by consumption demand shocks. What does the model imply about the cyclical behavior of the real wage and employment? Does this behavior seem consistent with the observed data discussed in class?