Econ 3510

Problem Set #6

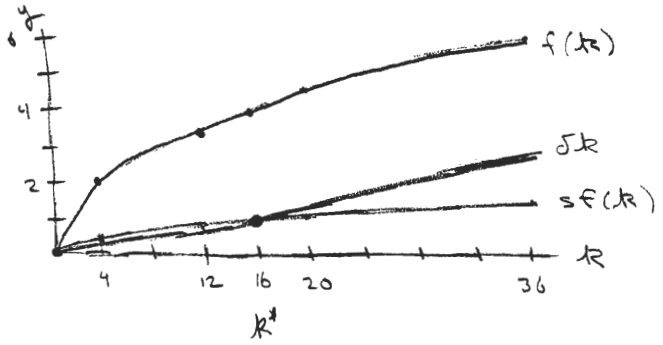
Answer Key

1. $y = k^{1/2}$ $\quad c = (1-s)y$ $\quad i = sy$ $\quad \Delta k = sf(k) - \delta k$

| $k$ | $y$ | $c$ | $i$ | $\delta k$ | $\Delta k$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 2 | 1.6 | .4 | .2 | .2 |
| 12 | 3.4641 | 2.77128 | .69282 | .6 | .09282 |
| 16 | 4 | 3.2 | .8 | .8 | 0 |
| 20 | 4.47214 | 3.57771 | .89443 | 1 | -.10557 |
| 36 | 6 | 4.8 | 1.2 | 1.8 | -.6 |

a) see table

b) $f(k+1) - f(k) = MPK$




c) $s = 0.20$

see table

d) $i = y - c$

see table

e) $\delta = .05$

see table

f) $\Delta k = i - \delta k$

see table

g) $\Delta k = 0$, $k^* = 16$, less than, falls

$y^* = 4$, $c^* = 3.2$, greater than, rises

h) $\Delta k = sk^{1/2} - \delta k = 0$

$sk^{1/2} = \delta k$

$k^{1/2} = \frac{s}{\delta}$

$$k^* = \left(\frac{s}{\delta}\right)^2 = k^* = \left(\frac{.20}{.05}\right)^2 = 16 \checkmark$$

2. $y = k^{1/2}$ $s = 0.20$ $\delta = 0.05$

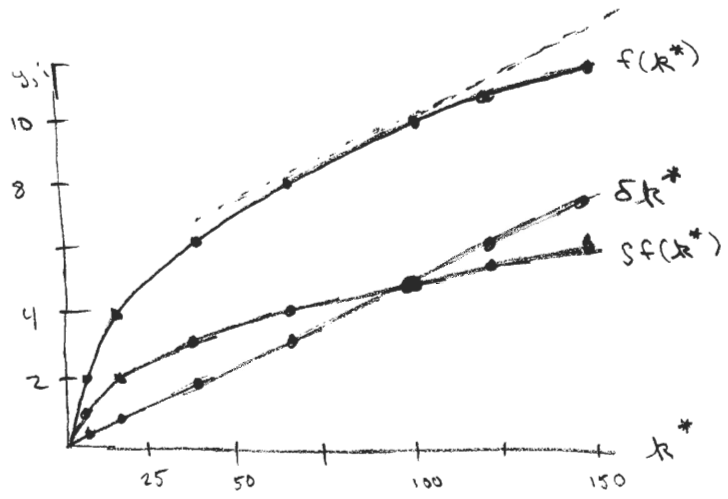
$c^* = f(k^*) - \delta k^*$

| $k^*$ | $f(k^*)$ | $\delta k^*$ | $c^*$ | $sf(k^*)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 4 | 2 | .2 | 1.8 | 1 |
| 16 | 4 | .8 | 3.2 | 2 |
| 36 | 6 | 1.8 | 4.2 | 3 |
| 64 | 8 | 3.2 | 4.8 | 4 |
| 100 | 10 | 5 | 5 | 5 |
| 121 | 11 | 6.05 | 4.95 | 5.5 |
| 144 | 12 | 7.2 | 4.8 | 6 |

a) $f(k^*) = k^{*1/2}$

$c^* = k^{*1/2} - \delta k^*$ see table

b)




$k^* = 100$

$k^*_{gold} = 100$

c) $\delta k^*_{gold} = 5$

$y^*_{gold} = 10$

$s = \dfrac{\delta k^*_{gold}}{y^*_{gold}} = \dfrac{5}{10}$

$s = 1/2$

increase

d) greater, increases

less, decreases

greater

e) $f(k^*_{gold}) = 10$

$f(k^*_{gold}+1) = 101^{1/2}$

$\therefore MPK = f(k^*_{gold}+1) - f(k^*_{gold}) \approx .0499$

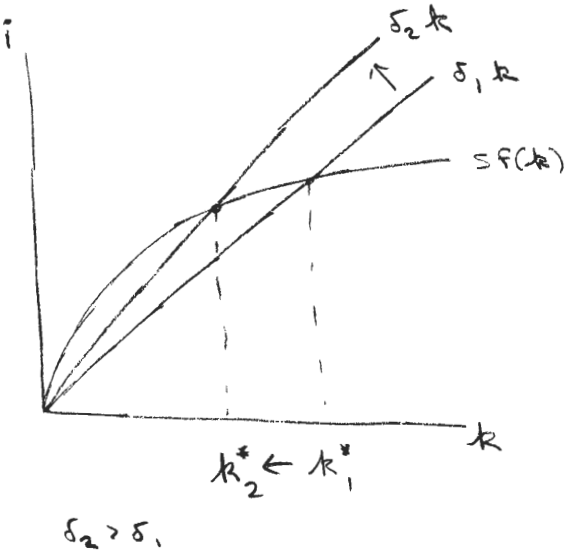
f) $s = 1/2$ equal to

$sf(k^*) = \frac{1}{2} k^{*1/2}$

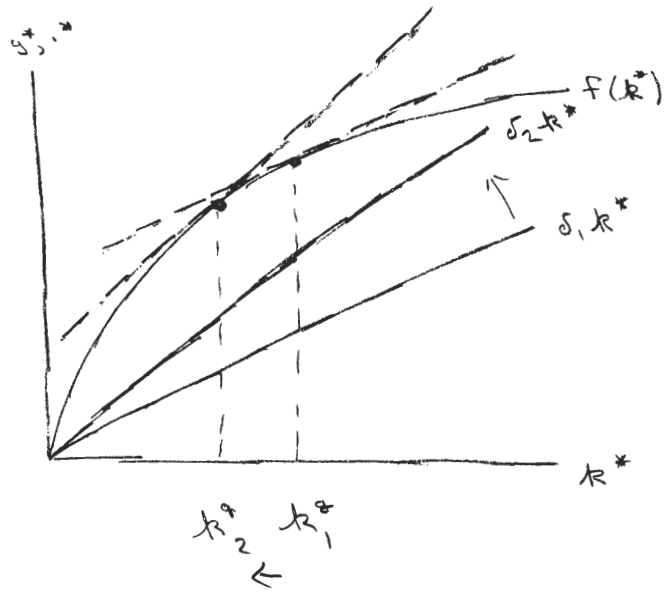
see table

3.

a)




$\delta_2 > \delta_1$




$k^*$ satisfies: $MPK = \delta$

if $\delta$ rises, MPK rises $\Rightarrow$ $k^*$ must fall

4. $Y = 10 \times K^{1/4} \times (E \times L)^{3/4}$ $\quad \delta = 0.10 \quad n = .04 \quad g = 0.02 \quad s = .128$

a) $$\frac{Y}{E \times L} = 10 \times K^{1/4} \times \frac{(E \times L)^{3/4}}{E \times L} = 10 \times K^{1/4} \times (E \times L)^{-1/4}$$

$$= 10 \times \left(\frac{K}{E \times L}\right)^{1/4}$$

$$\Rightarrow \quad y = 10\, k^{1/4}$$

b) $$\Delta k = s f(k) - (\delta + n + g) k = 0$$

$$s \times 10 k^{1/4} = (\delta + n + g) k$$

$$\frac{10 s}{\delta + n + g} = \frac{k}{k^{1/4}} = k^{3/4}$$

$$k^* = \left(\frac{10 s}{\delta + n + g}\right)^{4/3} = \left(\frac{10 \times .128}{.10 + .04 + .02}\right)^{4/3}$$

$$= \left(\frac{1.28}{0.16}\right)^{4/3} = 8^{4/3} = 16$$

$k^* = 16$

$y^* = 10\, k^{*1/4}$

$y^* = 20$

$c^* = (1 - s) y^*$

$c^* = 17.44$

$i^* = y^* - c^*$

$i^* = 2.56$

$\delta k^* = 1.6$

c) $\%\Delta \frac{K}{L} = \%\Delta \frac{Y}{L} = \%\Delta \frac{C}{L} = g = .02$

d) $\%\Delta K = \%\Delta Y = \%\Delta C = n + g = .06$