Economics 7720
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Suppose the economy is populated by a continuum of infinitely-lived identical households that rank alternative plans for consumption and money holdings according to the following lifetime utility function:

$$W = E_0 \sum_{t=0}^{\infty} \beta^t \left[a c_t^{1-b} + (1-a) m_t^{1-b} \right]^{\frac{1}{1-b}} \qquad 0 < a < 1 \quad \text{and} \quad b > 0$$

where c_t and m_t correspond to real consumption and holdings of real money balances at date t, and $\beta \in (0,1)$ is the subjective rate of discount.

After making consumption purchases, households allocate saving among investment in physical capital k_t, bonds b_t that return a nominal interest rate i_t, and money. The budget constraint takes the following form:

$$c_t + k_t + b_t + m_t = y_t + (1-\delta)k_{t-1} + \tau_t + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{1+\pi_t}$$

where δ is the rate of depreciation of physical capital, π_t is the inflation rate, and τ_t represents real government transfers (lump-sum). Consumption and capital goods are manufactured according to a standard neo-classical production function: $y_t = \mathrm{e}^{\nu_t} k_{t-1}^{\alpha}$. The exogenous variable ν_t is a mean-zero, serially uncorrelated productivity shock with variance σ_ν^2.

The government prints money at an exogenous fixed rate $\theta > 0$, that is, the nominal stock of money M_t evolves according to the following: $M_t = (1+\theta)M_{t-1}$. Assume that new dollars are injected into the economy via the lump-sum transfers τ_t.

(a) Write down the Bellman equation that characterizes the representative household's optimization problem. What are the state variables? What are the controls?

(b) Derive the set of first-order necessary conditions as well as the transversality conditions that together with the budget constraint fully characterize the optimal plans for the household's decision variables. List those variables that the household takes as given when formulating its decision.

(c) Derive the set of envelope conditions consistent with the household's optimization problem.

(d) Use your answers from part (b) and part (c) to derive the set of general equilibrium conditions that fully characterize the dynamic behavior of the following variables: $\{y_t, c_t, m_t, k_t, i_t, \pi_t\}$. Make sure that your equations do **not** contain elements that depend on the household's value function. Suppose you wanted to find the recursive representation of the model's solution. List those variables that would constitute the state or *predetermined* variables for that solution.

(e) Find expressions for the following variables in the steady state: $\{y, c, m, k, i, \pi\}$. Find an expression for $\frac{\partial m}{\partial \pi}$. Is it positive or negative?

(f) Discuss what is meant by the concept of superneutrality of money. Is this model consistent with the principle of superneutrality? Justify your answer.