

Recent empirical studies designed to measure the short-run impact of money on various economic aggregates have adopted a vector autoregression (VAR) framework (e.g. Christiano, Eichenbaum, and Evans (1999)).

Suppose that we are interested in determining the effects of an exogenous shock to monetary policy in the context of a multivariate VAR defined over the following variables: Y_t - log of real GDP, FF_t - the federal funds rate, NBR_t - log of nonborrowed reserves, P_t - log of the implicit GDP deflator, TR_t - log of total reserves, $PCOM_t$ - smoothed change in the index of sensitive commodity prices, M_t - log of M1. To this end, we construct the following fourth-order *structural* VAR:

$$FZ_t = B_1Z_{t-1} + B_2Z_{t-2} + B_3Z_{t-3} + B_4Z_{t-4} + u_t \quad u_t \sim iid(0, D)$$

where Z_t is a 7×1 vector containing the variables listed above in no particular order, and u_t is the 7×1 vector of corresponding *structural* shocks. The covariance matrix D is diagonal, ensuring that each shock is uncorrelated with all others. As an added restriction, assume that the diagonal elements of F are all one.

(a) The conventional method for estimating the parameters of a VAR is to apply ordinary least squares to each equation in the system. Discuss the empirical problems that would emerge if we were to attempt to estimate in the conventional way the parameters of the structural VAR given by $\{F, B_1, B_2, B_3, B_4, D\}$.

(b) In an attempt to recover structural estimates, suppose instead that we estimated the parameters of the following *reduced-form* VAR:

$$Z_t = A_1Z_{t-1} + A_2Z_{t-2} + A_3Z_{t-3} + A_4Z_{t-4} + \varepsilon_t \quad \varepsilon_t \sim iid(0, \Omega)$$

where Ω is a positive definite, symmetric matrix. How many prior restrictions would one have to place on $\{F, B_1, B_2, B_3, B_4, D\}$ in order to fully recover the structural estimates from the reduced-form estimates given by $\{\hat{A}_1, \hat{A}_2, \hat{A}_3, \hat{A}_4, \hat{\Omega}\}$? Describe a particular strategy for obtaining exact identification of the structural VAR and explain how you would recover the structural estimates. What specific prior restrictions would you impose on the values of $\{F, B_1, B_2, B_3, B_4, D\}$? Does the particular ordering of the variables appearing in Z_t matter? If so, discuss how you would order those variables and provide an economic rationale for your decision.

(c) Discuss some of the empirical findings in U.S. data that appear to be robust across a number of different identification schemes. Be sure to address the following points:

- What happens to FF_t and NBR_t following a contractionary policy shock?
- Is the data consistent with a liquidity effect? Explain.
- What happens to TR_t and M_t in the aftermath of a contractionary policy shock?

- Describe the effect of a contractionary policy shock on the behavior of real GDP. At what horizon (approximately) does the shock have its maximal effect?
- Describe the effect of a contractionary policy shock on the behavior of P_t . Approximately how long does it take for the shock to have any measurable effects?
- What is the justification for including $PCOM_t$ in the VAR? What problems emerge if it is excluded?