namic foundation. Whereas there existed no difference, from the kinematic standpoint, between the Copernican and the Ptolemaic systems, Newton, taking the standpoint of dynamics, decided in favor of Copernicus. For his theory of gravitational force offered to the latter view a mechanical explanation; whereas the complicated planetary orbits of Ptolemy did not fit into any explanation. If the question is how to provide both conceptions of the universe with an equal justification in terms of dynamics, then a general theory of gravitation has to be found, which explains the Ptolemaic as well as the Copernican planetary motion as a phenomenon of gravitation. Here lies the great mathematico-physical achievement of Einstein, in comparison to which Mach's thought appears merely as a first suggestion. Einstein has indeed found a comprehensive theory of gravitation, and only because of this discovery, which places his name in the same category with Copernicus and Newton, can we say that the problem of the relativity of motion has been brought, physically, to its conclusion.

Chapter 5 : GENERAL THEORY OF RELATIVITY

EVEN though the basic ideas leading to the general theory of relativity were clear to Einstein, the road to the complete theory was still long and laborious. Already in 1906, merely a year after the formulation of the special theory of relativity, Einstein had expressed the basic ideas of the new doctrine, going substantially beyond Mach. But the construction of the theory placed him before unsuspected mathematical difficulties. There was one period, in this path, when Einstein thought he had demonstrated the impossibility of a general theory of relativity. Only in 1915 did he succeed in completing the theory combining Mach's idea of the relativity of motion with the special theory of relativity into a completely new theory of gravitation, bringing thereby to a magnificent conclusion the era of classical physics. The news of Einstein's theory reached the public only in 1919 when an English expedition sent to observe an eclipse of the sun reported the first astronomical confirmation of his predictions.

In attempting to present Einstein's theory of gravitation, we must first get acquainted with the modification given by Einstein to Mach's idea. The idea of the relativity of force if stated in the form given by Mach, can be used only in connection with rotary motion. Einstein had to extend the idea in such a manner as to make it applicable to every motion. He achieved his aim through the so-called principle of equivalence.

We can clarity this principle by means of the so-called "box experiment" invented by Einstein in order to illustrate his ideas. Let us imagine a closed box of the size of a room, in which a physicist finds himself (Fig. 8). There is a spiral spring hanging down from the ceiling, to which an iron weight *m* is attached. The physicist

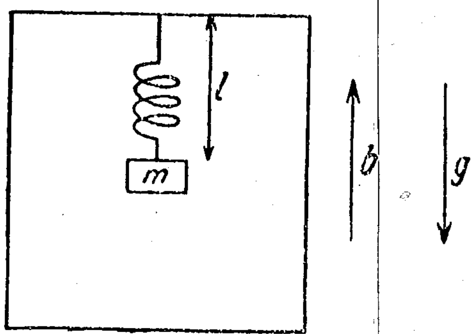

Fig. 8. Einstein's "Box Experiment"

has taken the measurement of the distance of the weight from the ceiling, i.e. of the distance to which the tension of the spring is adjusted.

The box has no windows. Were the box set in motion from outside, would the physicist notice the fact? Suppose that the box is being pulled up by a rope, like an elevator, in the direction of arrow *b*. Would the physicist inside notice it? Indeed he would be able to notice the change

in the interior of the box: the weight *m* would remain slightly behind the motion, on account of its inertia; the length of the spring would increase a little, accompanied by an increase in its tension. An accelerated or growing movement would thus result in a lengthening of the spring.*

Now, says Einstein, let us assume the physicist is aware of the lengthening of the spring; this is all that he observes immediately. Must he infer a motion of the box? Certainly, he can make this inference, for the motion of the box would produce this effect; but can this effect arise in no other way? If such a second cause is possible there is no necessity to infer a motion of the box.

Now, there exists indeed a second cause that could produce the same effect. If we assume that a great planetary mass is being gathered underneath the box, then it would produce a gravitational field. This field would act on the weight in the direction of the arrow *g* and pull it down. Again the physicist would observe an increase in the tension of the spring as well as an increase of its length *l*. From the observed lengthening of the spring the physicist, therefore, could just as well infer a field of gravitation below the box, as a movement of the box upward.

But is there no way of distinguishing between these two possibilities? Are there no other experiments enabling us to differentiate between a gravitational field and

*Were the motion uniform, that is, were the velocity of the box changeless, no expansion of the spring would take place. We must, therefore, keep steadily in mind, here and in the following, that the motion of the box is accelerated.

hand grows. One cause of the pressure is therefore contained in the bodily mass. We can increase the pressure also in a different way, without changing the body itself. If we visit one of those places of the earth, where the gravitation of the earth is stronger, then the body's attraction is magnified and its pressure on the hand is greater. In fact, there are such places. One could, for instance, descend into a deep mine pit; or one could go to the vicinity of a pole of the earth, which lies closer to the center of the earth, on account of its flattened shape, than do the middle or tropical zones. The variations of gravitation are not, to be sure, very considerable: they cannot be felt by the hand; more sensitive scales would have to be used. The scales in question could not be of the balance type, for the weights placed in one side would increase in weight just as much as the block of iron, with the result that the scales would indicate the same weight as before. One would have to use a spring scale, similar to those used in households; then, in the places located closer to the center of the earth, the spring will be more compressed.

The weight of a body is, therefore, different from its mass; it is the effect of attraction of this mass by the earth. At a great distance from the earth and other heavenly bodies, the weight of a body would be nil, while its mass would remain unchanged. On a large planet, such as Jupiter, all bodies are considerably heavier than on the earth. Our muscular strength would not be sufficient there, for instance, to lift a child from the ground, while

on a small heavenly body, such as the moon, we could pick up a grown-up person with great facility. We may define the mass, therefore, as that quality of a body, which determines its weight in a given gravitational field; the weight itself depends on that gravitational field.

The mass, if understood in this way, characterizes the body only with reference to the gravitational field and, therefore, in a rather one-sided manner. We shall call it "the heavy mass" of the body. Besides, there exists an entirely different effect of the mass, which leads us to the concept of "the inert mass."

Let us imagine a loaded railroad car. In order to set it in motion, a great force is required. This force is not directed, however, against gravitation, as the car rolls on horizontal tracks. It is the inertia of the load that opposes the motion. The applied force is, therefore, entirely independent of gravitation. In order to move the wagon on Jupiter, no more force would be required than on the earth, and vice versa; nor would this movement be easier on the moon. We designate as "the inert mass" that property which is determined by the opposition to changes in motion.

It is a fact of experience that the inert mass of a body equals its heavy mass. This is by no means a matter of course. This fact can be illustrated in the following manner.

Suppose that a log of wood and a block of iron lie on the large scales, and the two are found to be of equal weight. The log of wood is, of course, much larger. Now, hand grows. One cause of the pressure is therefore contained in the bodily mass. We can increase the pressure also in a different way, without changing the body itself. If we visit one of those places of the earth, where the gravitation of the earth is stronger, then the body's attraction is magnified and its pressure on the hand is greater. In fact, there are such places. One could, for instance, descend into a deep mine pit; or one could go to the vicinity of a pole of the earth, which lies closer to the center of the earth, on account of its flattened shape, than do the middle or tropical zones. The variations of gravitation are not, to be sure, very considerable: they cannot be felt by the hand; more sensitive scales would have to be used. The scales in question could not be of the balance type, for the weights placed in one side would increase in weight just as much as the block of iron, with the result that the scales would indicate the same weight as before. One would have to use a spring scale, similar to those used in households; then, in the places located closer to the center of the earth, the spring will be more compressed.

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Suppose that a log of wood and a block of iron lie on the large scales, and the two are found to be of equal weight. The log of wood is, of course, much larger. Now, both things are delivered, one after the other, to a railroad car; then we investigate whether it is equally difficult to set them in motion along the horizontal tracks. This is not a matter of course; one could surmise that the great wooden log would show more inertia-resistance than the small iron block, for their weight, or their pressure on the understructure, does not enter here into consideration. But experience instructs us that there is no difference at all. Bodies of equal weight have the same inertia; the heavy mass equals the inert mass.

This result also explains the fact that, with the elimination of air resistance in the vacuum, all bodies fall equally fast. The heavier body has a stronger downward pull, but at the same time it has to carry a greater inert mass: that is why it does not come down quicker.

After these considerations, we may return to our starting point, the physicist in the box, who is in possession of two equally justifiable explanations of the meaning of his findings. The connection of this Einsteinian consideration with Mach's criticism of the problem of rotation becomes now clear. Here, too, we find the duality of explanations: the observed effect of forces is either due to the resistance of inertia or to an overflow of a dynamic gravitational field. Whereas the observed effect was, in Mach's case, the centrifugal force and the pressure against the railing of the merry-go-round, in Einstein's case of the box experiment it is the tension of the spring, and the lengthening of *l*. But now we recognize the advantage of Einstein's presentation: it allows us to discover the reason for the

double explanation. In the two interpretations of the box experiment we referred once to the inertia of the weight ***m***, the second time to its heaviness. That both conceptions lead to the same observable effect is ~~a result of the~~ fact that the inert mass and the heavy mass are equal.

Although the equality of the inert mass and the heavy mass was long known, nevertheless Einstein was the first man to recognize the basic significance of this fact. He realized that here lies the reason why the distinction between accelerated motion and gravitation can not be made and why the physicist in the box can not, therefore, determine whether he is moving upward in an accelerated motion or a gravitational field interferes from below. Hence Einstein calls both conceptions equivalent, and maintains that it is meaningless to look for a truth-distinction between them.

With this assertion the problem is given a truly Einsteinian turn. For, when the equivalence is conceived as completely as it is done here by Einstein, the concept is found to be much richer in content than is offered by the experimental demonstration of the equality of inert and heavy mass. It represents a general assumption about all natural phenomena. This equivalence is supposed to hold not only for the mechanical, but also for the electrical, optical and other phenomena; in all these cases, no difference is supposed to result, whether one speaks of an accelerated motion of the box or of a gravitational field. A far-reaching hypothesis is assumed with this: it intimates nothing less than that the electrical, optical and other phenomena are to be included under the general theory of gravitation, that gravitation plays the same role in the doctrine of electricity, of optics, etc., as in mechanics.

I say that this is a truly Einsteinian turn. The physical depth of Einstein's ideas can be, indeed, comprehended only when one realizes how this method of reasoning is employed in his basic assumptions. This was the case in the special theory of relativity. It was known that several important attempts failed to confirm the existence of ether; Einstein concluded from this that, in general, no similar attempt can do better, no matter what means are used. The principle of equivalence reveals the same attitude. It is known that mechanical phenomena manifest no distinction between accelerated motion and gravitational field; Einstein concludes that this applies equally to all other phenomena. From the standpoint of logic, one cannot speak here of an inference, for this far-reaching assumption cannot be logically demonstrated by means of the scantily available facts. Rather. we have here a typical procedure in physics, that of the formation of a hypothesis; although a more extended assumption cannot be logically justified, nevertheless it is made in the spirit of a conjecture. There seems to exist something like an instinct for the hidden intentions of nature; and whoever possesses this instinct, takes the spade to the right place where gold is hidden, and thus arrives at deep scientific insights. It must be said that Einstein possesses this instinct to the highest degree. His assumptions cannot be justified in a purely logical way; yet they intro-

duce new ideas quite in the right place. That the place is right, can be readily recognized when gold lies in front of us. In physics, too, there is subsequent justification; for it is possible to perform experiments which later verify the new hypotheses. Thus it is possible to perform experiments testing Einstein's assumption that the electrical and optical phenomena are affected by gravitation. Such experiments have been made, and they have confirmed Einstein's hypothesis in a decisive way.

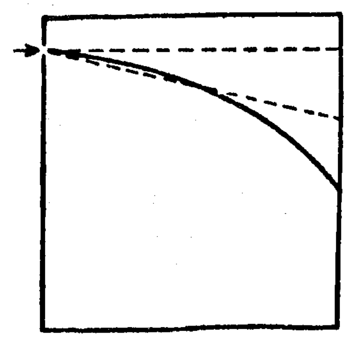

Fig. 9. The Curvature of Light-Rays in Einstein's Box

We shall elucidate this characteristic trend of thought by applying it to a certain example, namely, to the connection of light and gravitation. For this purpose, we turn once more to the box in which the physicist performs his experiments without being able to distinguish between acceleration and gravity.

Let us assume that the box is at rest (Fig. 9). In a side wall there is a small hole through which a ray of light shines in; it follows a straight, horizontal line in the dust of the air (represented by the dotted line of the figure). If the box is now set in uniform motion, the line changes: whereas the light entering through the hole reached previously exactly the opposite point on the wall, now that the box moves up the point of illumination goes further down, away from the ceiling. The ray is seen now as a sloping line, though still running straight. Next, let us imagine that the box moves upward with acceleration. The farther down sinks the ray, the faster goes up the box, so that the ray takes the distorted form of a curved line (see the solid line). In the dust of the air, it would be seen in the shape of a water jet spurting sidewise from the pipe and flowing down in an arc. This experiment cannot, of course, be actually performed, for the simple reason that light propagates so fast that, in contrast to it, the spatial displacement of the box in the same period of time amounts practically to nothing; no change in the ray could be actually observed. Our experiment is supposed to be merely "mental", intended to clarify the principle.

Let us now turn to Einstein's principle of equivalence. Einstein maintains it is immaterial whether we consider an accelerated motion or a gravitational field. It follows: As the curvature of the light rays occurs in the case of accelerated motion, so it must occur also in a gravitational field. The surprising conclusion results immediately from the principle.

We are facing here an entirely new consequence of

Einstein's theory of gravitation. The assertion is of a far-reaching significance. According to it, light does not propagate in open space in a straight line when it comes within the sphere of the attraction of masses; on the contrary, it follows a curved path not unlike that of a flying missile. This contention could be examined astronomically in repeated observations since Einstein deduced it for the first time from his theoretical considerations; and it has been confirmed to its full extent. Such observations not only require great precision but they can be made only during a total eclipse of the sun; elaborate preparations are therefore demanded of the astronomer who wishes to check Einstein's effect.

Einstein has drawn still another conclusion from his principle of equivalence, which concerns the behavior of clocks within the field of gravitation. By calculating certain deviations of the clock for the accelerated motion of the above mentioned box and by transferring the results to gravitational fields, he concluded, on the basis of considerations similar to those just outlined, that a clock, subjected to the influence of a strong gravitational field, would become slow. This effect cannot be demonstrated, of course, on ordinary clocks, as all watches and even the finest chronometers are still too inexact to be used for measuring these small retardations. But the physicist knows another kind of watches the precision of which transcends by far anything of human making: they are the individual atoms of which all substance is constructed. Let us describe briefly the plan for the demonstration of Einstein's doctrine, based on this effect.

Since the investigations of the last decades, it has become known that the atom is not a uniform body, but consists of two distinct kinds of material, the positively charged nucleus and the negatively charged electrones; the heavy but very small nucleus stands in the middle, while electrons revolve round it in their elliptical course. On account of this circular movement of the electrons, the whole atom can be conceived as a clock, in which each revolution of an atom corresponds to one turn of the hand and constitutes a unit of clock-time. Now, the revolution of electrons can be measured very exactly, insofar as it manifests itself in the number of vibrations of the light emitted by a circulating electron. Almost everybody has occasionally observed how a gas-flame becomes colored once salt gets into it; ordinary cooking salt colors the flame yellow, because it contains sodium; potassium colors the flame violet, etc. This coloration is due to the fact that the atoms of basic elements are "stimulated" by the flame and emit light the vibrations of which depend on the number of electronic revolutions, manifesting themselves in the color of the light. The exact estimation of the color is done by means of so-called spectral lines which are observed and photographed in an extremely delicate apparatus, the spectrometer. This apparatus splits every light into its component parts, so that white light is transformed by it into a "spectrum" resembling the color sequence of the rainbow and extending from red to orange, yellow, green, blue, and violet. The lights of

the radiating atoms, on the contrary, are marked in fine but sharp transverse lines, separated from each other, and each appearing in one definite color.

Einstein maintains that such an atomic clock manifests retardation in a gravitational field. A very strong gravitational field, a much stronger one than anywhere on the earth, exists on the sun, for the mass of the sun is by far greater than that of the earth. The atmosphere of the sun consists of incandescent gases; as the conditions prevailing there resemble those within the gaseous flame, atoms are aglow. In fact, with the help of a spectral apparatus, it is possible to recognize, as spectral lines, the colors emitted by individual elements of the sun and to measure the number of their vibrations. If the individual atoms are really somewhat retarded in their motion by the gravitational field of the sun, then the spectral lines arising in them must occupy a slightly different position in the spectrum than the lines arising in the earthly sources of light. They must shift in the direction of the lower number of vibrations, that is, toward the red end of the spectrum. One speaks, therefore, of the red shift of the spectral lines, observed in the sunlight.

The experimental test has encountered great difficulties at first, insofar as it deals with an extremely small deviation and the calculated effect lies just on the borderline of the measurable. But recently, very precise measurements have satisfactorily confirmed Einstein's findings. The astronomer, E. Freundlich, in order to

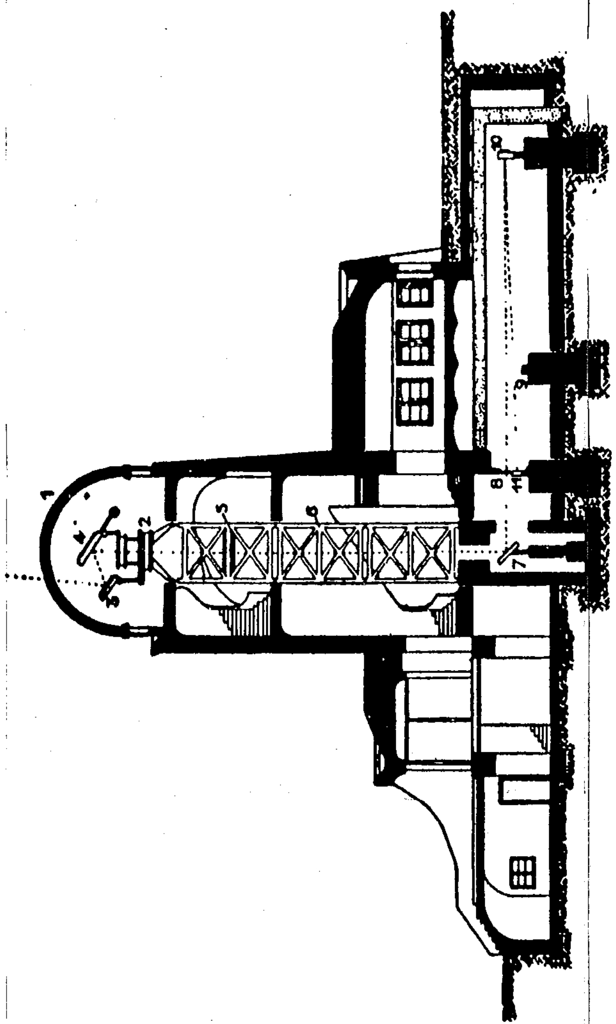

Fig. 10. *The Einstein Tower in Potsdam*

1. Cupola. 2. Revolving style for the mirror. 3. Coelostat. 4. Counter-mirror. 5. Objective. 6. Wooden scaffold. 7. Steering mirror. 8. Slot. 9. Prism apparatus. 10. Diffraction grating. 11. Photographic camera.

reach a conclusive demonstration of this, has built in Potsdam the Einstein tower (shown in Fig. 10), a structure combining to perfection every astronomical and physical contrivance. The tower has a lens (5) in its cupola, into which the light of the sun is directed from a side (mirror system, 3, 4), so that the tower as a whole forms a single large telescope. At the foot of the tower the light is caught (7) and directed toward a huge spectral apparatus. A space several meters long (8-10), which is completely shut off from the surrounding world, forms the interior of the apparatus. At 8 the light enters through a slot; and at 10 is found the most valuable instrument of the whole arrangement, the diffraction grating, consisting of a slightly curved metallic mirror with innumerable and extraordinarily fine scratch-lines. It splits light into its constituent colors and reflects it back to 11, where it is reproduced on photographic plates. The final measurements of the red shift are supposed to begin soon.*

Finally, we wish to mention, in this connection, the third astronomical test found by Einstein for his theory. With the mathematical elaboration of the theory, it became clear that the planetary movements followed a much more complex law than taught by Newton and

* The experiments in the Einstein tower could not be continued since Professor Freundlich was forced to leave Germany when the Hitler government came into power. The Einstein tower was given a new name and is now used for purposes which the Nazi government deems less dangerous for the German race. Up to the present time a definitive clarification of the red shift of spectral lines in the sun has not been given. (*Translator's note.*)

believed since his days. Newton's doctrine, to the effect that the sun attracts the planets with a power decreasing in proportion to the square of the distance, was shown by Einstein to be only approximately correct. It must be replaced, for more exact purposes, by a different law. Whereas every planet, according to Newton, describes an ellipse around the sun, it follows from Einstein's law that, though this ellipse is indeed described, nevertheless it is accompanied by another rotary movement: the ellipse, as a whole, revolves around the sun in the course of centuries. This rotary movement must be strongest for the planets in the neighborhood of the sun. The astronomers had noticed since the middle of the last century, that the planet Mercury shows certain deviations from its course: its ellipse actually executes a rotary movement of the kind. This was found in the lateral retrocession of one of the extreme orbital points, the perihelion. This so-called perihelion movement of Mercury amounts to only 43 seconds of the arc per century. Yet the astronomers were unable to find a satisfactory explanation of the fact. Einstein's law gave an explanation of this rotation of the ellipse.

The coincidence of theory and observation has, in this case, remarkable force of persuasion. It would not be surprising, if a theory devised originally for the explanation of the perihelion movement were to determine correctly the amount of this deviation. However, Einstein's theory has arisen from entirely different grounds. It is based on ideas concerning the relativity of motion, the

equivalence of gravity and acceleration; and all its constructions are made in the pursuit of this program. It was, therefore, highly surprising that Einstein, after being informed at a rather late stage of his ideas of the fact of the perihelion movement of Mercury, subjected his theory (rooted in entirely different sources) to the test of whether or not it will give an answer to this question. And when the long known amount of 43 seconds of the arc was deduced from his theory, he had every right to regard this unexpected coincidence as an excellent confirmation of his assumptions.

We have described in the preceding pages the astronomical consequences of the theory of relativity in such detail, because we are interested in showing that facts of observation have been the ultimately deciding factors in the acceptance of Einstein's theory. This is its strength; for, in the last analysis, the final confirmation of physical ideas can be given only by nature itself. Were it merely the question of creating a picture of how to make intelligible the inner workings of nature, physics would be a very simple science. Explanations are found altogether too easily, when imagination is given a little rein. But it is truly an art to find explanations from which new facts follow and which can be confirmed by experiments. This applies, above all, to the numerous inventors who still occupy themselves with the problem of ether and who still look around for ideas as to how to reconcile the contradictory properties of such ether. Such ideas can always be found; but they lack the force of conviction, because their authors do not succeed in getting new experimental results from their theories. It is easy to devise a theory of ether, capable of accounting even for the curvature of light and the red shift; there is no trick to it after these effects have been discovered by Einstein. Whoever believes firmly in the existence of ether should take example from Einstein and *predict* effects capable of experimental proof. But as long as this does not occur and only the prenomena predicted by Einstein are observed, so long shall we adhere to Einstein and to his theory of gravitation, which is also a theory of the relativity of motion.

We do not wish to attempt presenting the mathematical structure of Einstein's theory. Nobody will doubt our words that, mathematically, it is an exceedingly intricate matter. Einstein aimed to find a general concept of gravitation that would fit all the different descriptions which could be given for the state of gravitation. For this purpose, he had to introduce in physics a new mathematical method, the so-called tensor calculus. We are reminded here of Newton's case who, in a similar manner, had to develop a new mathematical method, that of the differential calculus, on which to construct his theory of gravitation. However, whereas Newton had to invent, at that time, the method of calculation himself, Einstein was fortunately able to utilize for this purpose the mathematicians' works which were already available. The essence of the new method of calculation resides in two basic concepts, the *invariant* and the *co-variant*. The field

of gravitation is a co-variable magnitude. If one passes from one frame of reference to another, this magnitude changes, varies with—and this is the meaning of the word "co-variant'. Nevertheless, one should not believe that the objective meaning of the knowledge of nature would be eliminated thereby; for all such descriptions given in terms of different frames of reference signify merely different ways of expression, enabling us to comprehend the true character of nature. It is something like the way in which one can express thought in German, English, French, etc.; the language may be different, but the mental content is the same. Similarly, the presentation of the state of gravitation in the world can be made in different languages, depending on the chosen frame of reference. But all these descriptions refer to one and the same objective state. This state is the invariable, the unchangeable. The peculiarity of the mathematics of relativity is perhaps best expressed in this pair of concepts, the invariant and the co-variant. The co-variant stands for the manner of description; the invariant, for the common state arrived at from all the various descriptions.

It is important to make this thought clear. It is occasionally attempted to present Einstein's theory in the simple sentence that everything is relative. But Einstein has not made everything relative. Only some things have become relative, particularly things previously regarded as absolute verities. On the other hand, the theory has made only clearer the things which are true regardless of the arbitrariness of descriptions. By pointing out the arbitrary additions made by man in his description of nature for what they are, Einstein's theory has made objective truth stand out more visibly than ever. Thus, the theory of relativity represents the highest level on the road to an exact knowledge of nature, along which the natural sciences have proceeded for centuries with so much success.

Chapter 6 : SPACE AND TIME

IN THE preceding chapters we have described the physical side of the discoveries connected with the theory of relativity. In doing so, we put a special emphasis on factual foundations, that is, on the data of observation and experimentation, which gave rise to the bold conclusions drawn by Einstein. In this last chapter, we intend to consider the other side of the problem, dealing not so much with physics as with another realm, that of philosophy. Our theory will appear, in this light, no less important and significant. We encounter here the thoughts which made the theory of relativity famous in wide circles, which distinguish it from other physical theories and secure for it a prominent position within the modern philosophy of nature. It is the revolution of our ideas concerning space and time, to which we turn with this analysis.

As far as time is concerned, a substantial part of the new ideas has already been presented in the chapter on the special theory of relativity. The foremost place is occupied here by the relativity of simultaneity; it maintains that the time-order of events separated by distance is arbitrary within certain limits. It must be stressed once more that the events in question must be widely separated in space. We have found that the time-order of such events is not accessible to direct observation. As observers, we can be in the neighborhood only of one of the events; a signal must be sent from the other event, which thus notifies us of the event's existence. If we wish to be informed as to the time at which it occurred, we must resort to calculation; for that we must know the velocity of the signal. Yet we have found that it is impossible to measure the velocity, unless we have already established simultaneity; for such a measurement requires two clocks, correctly set and placed at different localities. The argument thus runs in a circle, one premise presupposing the other; and its solution consists in abandoning the objective meaning of simultaneity. Simultaneity cannot be known, it must be defined, and this definition will be arbitrary to a certain extent. If cannons were fired on two distant mountains at the same time, I should hear the two reports simultaneously only if I were standing in the middle of the distance. I then could assert also that the two discharges did not occur simultaneously but in succession; and that could be justified by ascribing to sound waves a greater speed in one direction than in the other. I could then consider, quite arbitrarily, one or the other discharge as the earlier. Such an assertion would never involve me in contradiction; for I shall always be able to account for my observation: namely, that I hear the two reports simultaneously in the middle of the distance.

Here lies one of the deepest thoughts of the theory of relativity. We shall regard as true whatever we observe immediately; no theory can put out of existence whatever

our senses teach us. An unconditional respect for the evidence of the senses, of experience, constitutes the basic principle of the theory of relativity. This is supplemented, however, by the clear realization that the power of human observation is limited. Only a small portion of the world-space can be mastered by the senses; whatever happens beyond it, must be deduced by reflection. This is where reasoning comes in; by its force our knowledge expands beyond the narrow horizon of vision and opens up before us the gates of distant worlds. When we declare that we see the stars, this is a very inexact way of expression; we see directly only the light penetrating our eye. If we proceed from the experience of brightness, occurring here, to the statement that there are stars far away, we are compelled to draw an inference; and this inference cannot be drawn without some arbitrariness. One part of this arbitrariness is represented by simultaneity. The way we define it can change our system of thought, but it cannot change the observed facts themselves; that is why all these different descriptions are equally true and equally justified.

The relativity of simultaneity has a peculiar consequence, as far as the measurement of space is concerned. We shall make this clear by means of an instructive example. For this purpose we consider an apparatus, well-known in photographic practice, the so-called focal-plane shutter.

Most photographic cameras are equipped with a shutter mounted between the lenses; but all these shutters prove to be inadequate for the photography of fast moving objects, because their exposure time cannot be made short enough. A focal plane shutter is used, therefore, for very short exposures. In such a camera there runs vertically outward, close to the film, and therefore practically in the focal plane, a rolling curtain with a horizontal slit in it; the various parts of the film receive light only as long as the slit passes them. The time of exposure is, therefore, extremely short. But at the same time a peculiar fault creeps in: the individual sections of the plate do not receive light all at the same time, but only one after another, and as the object moves while being photographed the individually illuminated sections do not

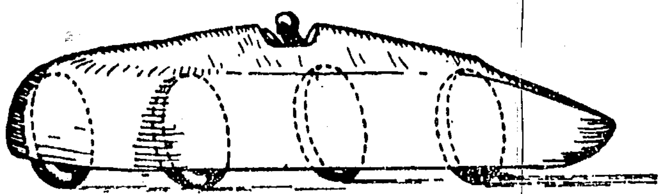

Fig. 11. Major Segrave's 1,000 Horsepower Auto at Full Speed

represent strictly simultaneous states of the object, but successive states. The object cannot change very much, however, in that brief period of time; nevertheless, a certain distortion of the picture does occur. This can be well observed on the wheels of a fast moving automobile, since they assume the shape of a somewhat crooked ellipse with a forward tilt (Fig. 11).

A similar distortion occurs, according to Einstein, when one wants to determine the shape of moving bodies.

The difficulties found here were not seen at all before Einstein. For if one observes a moving body from a frame of reference at rest, the moving object is "photographed", so to speak, from a position at rest; and then the image is examined. The moving body appears to an observer at rest as a sequence of such instantaneous snap-shots. At this point the relativity of simultaneity comes into consideration; events which are conceived as simultaneous for one definition of simultaneity, represent a sequence of time for another. The significance of this, as far as pictures of moving bodies are concerned, is as follows: what is instataneous photography for one temporal system, is a photography by focal plane shutter for another. The shape of moving bodies varies according to the definition of simultaneity. There are no true shapes of moving bodies: all shapes obtainable in this way are equally true.

This is Einstein's theory of the change in the form of moving bodies. The comparison with a photography by focal plane shutter represents the nature of this theory extremely well. The only difference consists in that Einstein's focal plane shutter would have to run faster than light. It therefore cannot be actualized by such an apparatus as a photographic shutter. On the other hand, it follows from this fact that Einstein's "distorted snapshots" are not "false"; they can just as well be considered as strictly instantaneous snapshots. This result does not hold for ordinary photography by focal plane shutter; pictures so obtained must rightly be called distorted.

Our reflection shows us that space-measurement depends on simultaneity. This idea can be expressed mathematically by bringing together space and time into a four-dimensional structure, into a space-time manifoldness. Strangely enough, this procedure which appears simple and harmless to the mathematician, has given cause for great surprise and for bewilderment to others. Many a reader of books on relativity thought that space was thereby transformed from a three-dimensional structure into a four-dimensional one; and he then attempted in vain to conceive the fourth dimension of space. He may have argued in this way: Imagine three sticks of wood meeting together at one point under right angles, like the length, width and height of a room. These are three dimensions of space; is there any room for the fourth one? How is it possible to pass the fourth stick through the point, so that it too would form right angles with the others? The author too cannot visualize how it would run; but the theory of relativity never asserted anything of the sort. It asserts merely that time should be added, *as time*, to space; and this is something entirely different. We may imagine it this way: Three numbers are needed to determine a point in space. Suppose a lamp hangs in the room. How can we determine its place? We measure its distance from the floor, from the back-wall and from the side-wall; these three figures determine its position in space. The three numbers are called co-ordinates. The room is three-dimensional, because three figures are needed for statements of the kind described. If we want to determine not a point in space but an event,

we require another figure, namely, the statement of time. Suppose that we switch on the light for a second and produce a flash of light; this is an event. It is completely determined if we know the three numbers defining the position of the lamp and, in addition, the fourth number defining the time of the light-flash. Insofar as there are four figures, space and time together are called a four-dimensional manifoldness. This is the whole secret. Unfortunately, this simple circumstance is often depicted in a most obscure language.

Whatever new is asserted by the theory of relativity about the space-time manifoldness, is illustrated much more comprehensibly and clearly in our picture of the focal plane shutter. It shows that the measurement of space is dependent on the measurement of time. This is, of course, something very new and profound; but it does not deprive time of its specific temporal character. Rather, it must be said that only the theory of relativity has discovered and formulated the peculiar distinction of time and space. The philosophical investigation of the theory of relativity has shown that time is something even more profound than space, that it is connected with the deepest principle of all knowledge of nature, the law of cause and effect.

If we now turn to the problem of space, we find here ideas going farther back than the relativistic doctrine of time. For what Einstein teaches about space and geometry, has been prepared, on the mathematical side, one hundred years ago. These ideas are connected with the so-called non-Euclidian geometry. The geometry studied by us in school goes back to the Greek mathematician, Euclid; it has been taught for two thousand years in the form originally given by him. Only within the last century a new kind of geometry was discovered by several mathematicians, among whom Riemann is the most important. This geometry appears at first glance totally unreasonable and nonsensical, insofar as it contains such sentences as that the three angles of a triangle are together more than 180°, or that the circumference and diameter of a circle do not stand in the relationship $\pi = 3.14$. A more exact examination, however, proves it to be a completely correct and permissible mathematical system, to which one has only to get used.

The non-Euclidian geometry may be conceived simply as a play with concepts which, though logical in themselves, have no significance beyond that. It seemed in fact that real space, the space of things and bodies of the universe, followed the laws of old Euclidian geometry. These laws were always taken as basic, whenever houses and streets were built, or areas measured for topographic maps, or cosmic distances calculated. But already the discoverers of non-Euclidian geometry asked themselves the question as to whether Euclid's laws are strictly true; possibly, they thought, more exact measurements may bring to light deviations corresponding to non-Euclidian geometry. They knew full well that such deviations can be expected only for very large dimensions. The great mathematician, Gauss, undertook therefore to measure

a triangle of large size. The corner-points of his triangle were formed by three mountains: Brocken in Harz, Inselsberg in the Thueringian forest, and Hohenhagen near Goettingen. The summits of these mountains were almost at the limit of visibility from each other, if telescopes were used. Gauss measured the three angles enclosed by this triangle and inquired whether their sum differed from 180°; however, there was no noticeable deviation. Nevertheless, some mathematicians and physicists believed ever since then that some day a deviation may be revealed in still larger triangles by means of more precise instruments.

The relations governing space, in that case, can be elucidated if we take as our starting point the corresponding relations in two-dimensional surfaces. It is found that the laws similar to those holding for non-Euclidian geometry of three-dimensional space actually apply to such two-dimensional structures as curved surfaces. At the same time, let us depict much greater deviations than those assumed in Gauss's experiment; it then will be easier to visualize the relations to be considered.

Let us imagine beings living on the surface of a globe, for whom nothing exists outside this globe-surface. In their world, there would not be any tunnel going through the globe; nor would it include things stretching away from the globe, such as trees or towers. Everything is flat for them, embedded completely in the surface of the sphere, including the beings tehemselves. Now the question arises: would these beings be capable of noticing that they live on a curved surface?

The answer to this question is by no means self-evident. We notice the curvature of the surface of the earth mainly because we observe phenomena outside the two-dimensional surface. When we observe the curvature of a hollow in the ground we *sight* across it, i.e., we compare its form with the course of light-rays; we see the curvature of the hollow merely because light is not confined to the curved surface but freely permeates the three-dimensional space. But in the two-dimensional world as conjectured, light-rays would glide along the surface; therefore no curvature would be noticed by sighting. And yet there would be other ways to recognize the curvature.

Suppose that those living beings undertake surveying; they draw figures in the sand and measure them with yardsticks. They draw a circle around the north pole of the globe, for instance, a circle corresponding to 89° of northern latitude. Then they measure the circumference of the circle, using the yardstick. Finally, they measure the diameter of the circle; but what will they measure as diameter? Certainly not the "true" diameter traversing the interior of the sphere, along the chord; for they cannot leave the surface of the globe, and there does not exist anything for them outside the surface. Consequently, they will take for diameter the curved line running from one point of the circle by the north pole to its opposite point. This line will appear straight to them, because, in

following it with the eye, they see the opposite point, insofar as light moves along the contour of the globe. But, if they measure the length of this line by using the yardstick, and then divide the circumference of the circle by the figure obtained for the diameter, they will get a smaller number than $\pi = 3.14$, as the measure of the diameter is too large. By the results of these measurements they will know that they live on the surface of a globe.

Now let us describe the corresponding situation for three dimensions. Suppose there is a large sphere of iron sheet, about the size of a house. There is an iron scaffold inside. A man climbs on it; he can climb also the outer surface, where there are handles and steps to cling to. He measures the circumference of the sphere with a yardstick and then the diameter in a similar way, climbing along one of the girders. Finally, he divides the figures and gets a smaller number than $\pi = 3.14$.

The result was easy to understand in the case of two dimensions. The surface was conceived as curved or bent in the third dimension, as a sphere's surface must be. But for the case of three dimensions, this answer is no longer possible. There is no room for curving the three-dimensional space. How shall we then interpret the result? Nothing remains for us to do but to admit that we live in a non-Euclidian space. Those experiences in measuring are what would be noticed in such a space as space-curvature. Furthermore, we must keep in mind that the described two-dimensional creatures would have no other way of visualizing the curvature of their two-dimensional space; they cannot speak of its bending in the third dimension. The deviation from normal measuring conditions is just what one would experience inside a non-Euclidian space.

We cannot go here any further into the problem of visualizing non-Euclidian space; for a more detailed treatment of these questions, we must refer the reader to the author's *Philosophy of Space and Time*,* which in general must be consulted for a more extensive explanation of the thoughts contained in this book. There we discuss, in particular, the question of the relativity of geometry; it appears, namely, that all geometrical measurements imply an uncertainty similar to that of the relativity of motion, and that measurements of the objective geometry of space presuppose a special sort of definitions which we call coordinative definitions. This question is connected with the question of whether there exists a Euclidian interpretation of measurements as described. Here we must face the question as to how Einstein came to apply non-Euclidian geometry to his theory of gravitation.

We have already pointed out in Chapter 3 that watches and yardsticks have no independent significance, according to Einstein's conception, but change in a particular way and are adjusted to the geometry of light. But even light is not the final thing; for it, too, is subjected to the guiding power of gravitation. It may be well to remind

*H. Reichenbach, *The Philosophy of Space and Time,* English translation, Maria Reichenbach and John Freund, Dover Publications, Inc., New York, 1957. Cf. also H. Reichenbach and E. S. Allen, *Atom and Cosmos: The World of Modern Physics,* Ridgeway Books, Philadelphia, 1933.

here of the argument contained in Chapter 5, according to which light conforms to the gravitational field. Gravitation is the primary effect of the masses filling space; it is the guiding power to which light, yardsticks and watches conform. The simple relations of spatial measurement, as formulated in Euclidian geometry, are valid only in the absence of a gravitational field, that is, at great distances from the star masses. In the vicinity of such great masses, on the other hand, space is warped, so to speak; it assumes curved forms and follows strange laws, as given in non-Euclidian geometry. The deviation from Euclidian relations is always, to be sure, very small, so small, in fact, that it cannot be demonstrated by means of ordinary measuring devices. This is the reason why it passed so long unnoticed. Even such measurements as those of Gauss could lead to no success, because they invariably dealt with too small distances. The deviations manifest themselves only in cosmic distances; and it is the course of heavenly bodies and of light-rays between them that betrays the non-Euclidian nature of space. And there, in the wide stretches of the universe, we find, indeed, quite substantial changes of geometry.

The most perplexing thing of it all is that the space of the universe must now be considered as finite. This does not mean that the masses of the stars alone are finite; it means that space itself is limited. We can visualize this in the following manner. If a ray of light is sent out in a straight line, it returns after a certain time from the opposite side, not unlike a ship sailing steadily west but returning to the port of departure from the other side. There is no unlimited extension in this space; all straight lines come finally to their source. Each star can be potentially seen twice, therefore, once from the front and the second time from behind, when we look at it about the universe. Unfortunately, no proof of this theory of Einstein can be given at the moment, for the road around the world is so long that the stars' light grows too weak to be observed. But even if we could see the light, there would be no way of recognizing the particular star. In the countless thousands of years required by light to go around the world, the star would have wandered far away and would occupy an entirely different position from its counterpart; as a result, we should not be able to recognize the two stars as identical.

Einstein's conception of gravitation as a "metric power", as a force determining the relations of spatial measurement, leads therefore to a far-reaching revolution in our knowledge of space. Apart from the novelty of the theory of a limited heavenly space, which signifies a turning point similar to that of the doctrine of the spherical shape of the earth, at the time of its promulgation, the method of dealing with the problem of space, applied in Einstein's theory, represents a new form of philosophical thinking. It follows the principle that statements concerning space are not to be separated from statements concerning bodies in space, that a space has no absolute significance apart from things and the laws of their mutual relations, a principle recognized before

Einstein only by Leibniz. This limitation of the concept of space to its bodily manifestations represents a key to the understanding of the meaning of geometry, a problem which, after the discovery of non-Euclidian geometry, could no longer be solved by Kant's doctrine of an apriori validity of Euclidian geometry. The apparent priority of the latter geometry, expressed in the fact that it controls all our spatial imagery, can be understood if we realize that the space-perception we possess has arisen historically from contact with things following the laws of Euclidian space. The solid bodies and sticks we work with comply so closely with the rules of Euclidian geometry that we do not notice any deviations from it; as a result, we have become so accustomed to the laws of Euclid that we regard them as absolutely necessary. The deviations pointed out by Einstein occur only in astronomic dimensions. Were we to live, however, in a world where the laws just described should hold in the dimensions of our daily environment — where, for example, the measured relations between circumference and diameter would differ from 3.14 — we should get accustomed also to these facts. We should find everything self-evident and natural. If a physicist came along and asserted the opposite, namely, that Euclidian geometry must determine all our spatial imaginations, we should answer him that he asserts the impossible; and his loudest opponents would be the very persons who defend today the apriori character of Euclidian geometry. The great achievement of Einstein consists in that his thinking is free from conventional ideas, that he did not hesitate to disregard the oldest laws of natural science, the laws of geometry, and to set new ones in their place. Though these new geometrical laws were recognized by other mathematicians before him, Einstein was the first one to take them down from the shelves of thought-possibilities and to apply them to physical science, to the description of nature. Such a scientific deed manifests boldness, reveals independence of thought; and we should not be astonished that it was difficult for all of us, and will be so for every one who hears of these ideas for the first time, to understand Einstein's theory.

Once more a chapter of our presentation ends with a Copernican turn. The first such turn was given by the demonstration of the relativity of motion; with this principle the step from the Ptolemaic world view to the Copernican one was repeated on a higher level, leading to a synthesis of both world views into one. In a similar way, the break with Euclidian geometry shakes the very foundations of our knowledge and signifies a transition to a knowledge of a higher kind, incomprehensible as this knowledge may appear at first view. But just as the Copernican worldview became at last generally recognized and a common property of all educated people, so will it be with the theory of relativity. One hundred years from now, the doctrine will be accepted as self-evident; and it will be difficult to comprehend why it encountered at first so much opposition. In Schopenhauer's words, "Truth is allowed only a brief interval of victory between

the two long periods when it is condemned as paradox or belittled as trivial." We who are permitted to see this period of victory with our own eyes may consider ourselves fortunate to witness the Copernican discovery of our age.