Homework 1

Economics 7110

Spring 2009

Middle Tennessee State University

This homework is intended to give you a review of some basic concepts in linear algebra and to introduce you to some of the features of `matlab`. It may be helpful to review the definitions of inverse, trace, and eigenvalues before starting the homework.

1. Go to our web page and copy the `matlab` file `ex1.m` from my directory to a new file in your directory called `hw1.m`. Then, modify your hw1.m file to answer the following questions.

   (a) In the code, create a 10×10 matrix defined by: $\mathbf{M} = [\mathbf{I} - (1/10)\boldsymbol{\iota}\,\boldsymbol{\iota}']$ where $\mathbf{I}$ and $\boldsymbol{\iota}$ have been defined in class. Then, show that: $\mathbf{M} = \mathbf{M}'$; and $\mathbf{M} = \mathbf{MM}$. That is, $\mathbf{M}$ is symmetric and idempotent.

   (b) Alter the code to compute two new matrices:

$$\mathbf{y} = \begin{bmatrix} 89.2121 \\ 89.8565 \\ 90.4337 \\ 89.6069 \\ 90.8523 \\ 89.1211 \\ 90.7312 \\ 89.9642 \\ 90.1163 \\ 88.8782 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 11 \\ 1 & 13 \\ 1 & 22 \\ 1 & 12 \\ 1 & 24 \\ 1 & 10 \\ 1 & 9 \\ 1 & 8 \\ 1 & 17 \\ 1 & 22 \end{bmatrix}.$$

   Compute $\mathbf{M} = [\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']$. Then, show that: $\mathbf{M} = \mathbf{M}'$; and $\mathbf{M} = \mathbf{MM}$. That is, $\mathbf{M}$ is symmetric and idempotent.

   (c) Alter the code to show: $(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X} = \mathbf{I}$ and $\mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$; the general property of inverse. Also, show: $\mathbf{X}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} = \mathbf{I}$ and $\mathbf{M}\mathbf{M}^{-1} = \mathbf{I}$.

2. Consider some more `matlab` questions. Enter the following new matrices:

$$P = \begin{bmatrix} .9 & .1 \\ .3 & .7 \end{bmatrix}, \quad y = \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \quad \pi = \begin{bmatrix} .5 \\ .5 \end{bmatrix}.$$

   Then, alter the code to produce the following quantities.

   (a) Compute $\pi_{1,1}P_{1,2}P_{2,1}P_{1,2}P_{2,1}$.

   (b) Compute $\pi_{1,1}P_{1,1}P_{1,1}P_{1,1}P_{1,1}$.

   (c) Compute $\pi_{2,1}P_{2,2}P_{2,2}P_{2,2}P_{2,2}$.

   (d) Compute $\pi' y$.

   (e) Compute Py, P^2y, and P^3y.

(f) Compute $y + .9Py + .9^2P^2y + .9^3P^3y + .9^4P^4y + .9^5P^5y + .9^6P^6y + .9^7P^7y + .9^8P^8y$.

(g) Compute $(I - .9P)^{-1}y$.

(h) Compute

$$\begin{bmatrix} 1.8 & 5.8 \\ 3.4 & 15.4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 5 & 25 \end{bmatrix}^{-1}.$$

(i) Using `matlab`'s `eig` routine, compute the eigenvalues and eigenvectors of P'. For help, enter `help eig` at the `matlab` prompt.

(j) Using `matlab`'s `trace` routine, compute the trace of P. For help, enter `help trace` at the `matlab` prompt.

(k) Show that $trace(P') = trace(P)$.

(l) Show that $trace(P'P) = trace(PP')$.

(m) Show that $trace(P+P) = trace(P)+trace(P)$. That is, trace is a linear operator.

(n) Show that $trace((\pi'P)'(\pi'P)) = trace((\pi'P)(\pi'P)')$.

(o) Show that $trace(P(\pi'P)'(\pi'P)) = trace((\pi'P)'(\pi'P)P)$.

(p) Show that $trace(P(\pi'P)'(\pi'P)) = trace((\pi'P)P(\pi'P)')$.

3. Consider some theoretical questions. That is, do not use a computer.

(a) Show that $\mathbf{MM} = \mathbf{M}$ where $\mathbf{M} = [\mathbf{I} - (1/10)\boldsymbol{\iota}\,\boldsymbol{\iota}']$.

(b) For any $\mathbf{X}$, show that $\mathbf{MM} = \mathbf{M}$ where $\mathbf{M} = [\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']$