Homework 4

Economics 7110

Spring 2009

Middle Tennessee State University

1. Consider the following model with external habit persistence. Current individual consumption depends on previous community consumption and on previous individual consumption. That is:

$$c_t = \lambda C_{t-1} + c_{t-1}$$

Let community consumption evolve by the following equation:

$$C_{t+1} = \mu + \phi C_t + c w_{t+1}$$

where $w_{t+1} \sim N(0,1)$.

   (a) Show how to map these processes into a first-order linear stochastic difference equation where $x'_{t+1} = [c_{t+1}, c_t, C_{t+1}, C_t, 1]$.

   (b) Suppose individual preferences are determined by the following quadratic form:

$$u(c_t, c_{t-1}) = (c_t - c_{t-1})^2$$

   and we wanted to calculate:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1})$$

   Find a reduced form formula for the expectation that can be used for calculation (or two equations that when solved can give the formula for the infinite discounted sum). Please specify, for example, the Y matrix.

2. Consider the "cake eating" problem of

$$\max_{\{c_t, w_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to $c_t + w_{t+1} = w_t$ where w_0 is given and is time zero wealth. Let $v(w)$ be the optimal value function.

   (a) Set up the dynamic programing (D.P.) problem. That is, write down the Bellman's equation and the critical elements needed for a D.P. problem.

   (b) Find the F.O.C.s, Benveniste-Scheinkman, and Euler equations.

   (c) Find the optimal solution by iteration.

3. Consider the model of exercise 3.1 on page 94.

   (a) Solve for the optimal policy by value function iteration.

   (b) Solve for the optimal policy by guess and verify with guess $v(k_t) = A + B \ln k_t$.

(c) Solve for the optimal policy by guess and verify with guess $v(k_t) = A + B \ln k_t + C \ln \theta_t$.

4. A monopolist faces an inverse demand curve

$$p_t = 1000 - Q_t,$$

and has cost function

$$c(Q_t, Q_{t+1}) = Q_t + .5(Q_{t+1} - Q_t)^2.$$

Given Q_0, the monopolist chooses a sequence $\{Q_{t+1}\}_{t=0}^{\infty}$ to maximize profits π:

$$\begin{aligned} \pi &= \sum_{t=0}^{\infty} \beta^t (p_t Q_t - c(Q_t, Q_{t+1})) \\ &= \sum_{t=0}^{\infty} \beta^t ([1000 - Q_t]Q_t - c(Q_t, Q_{t+1})). \end{aligned}$$

Basically, each period t the firm maximizes profits by choice of Q_{t+1} given Q_t and the value function.

(a) Formulate, in quadratic form, the Bellman's equation for the monopolist. That is, what is x_t and u_t? Also, what are Q, R, and W associated with the return written in quadratic form: $r(x_t, u_t) = -(x_t' R x_t + u_t' Q u_t + 2u_t' W x_t)$. Finally, what are the A and B matrices associated with a linear transition function of the form: $x_{t+1} \equiv g(x_t, u_t) = Ax_t + Bu_t$ (include a constant in x).

(b) Guess that the value function takes the from: $x_t' P x_t$. Then, solve or the optimal policy as a function of the undetermined coefficients P.

(c) Find equations that when solved will give the matrix parameters of the value function and thus verify your guess.