Related Samples (paired) *t*

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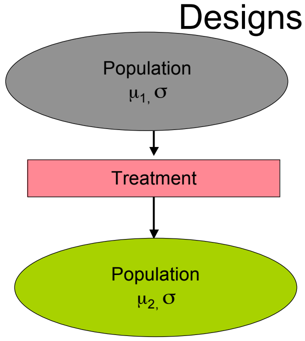
Background

Most typically in research we do not know either the mean or the standard deviation of the population.

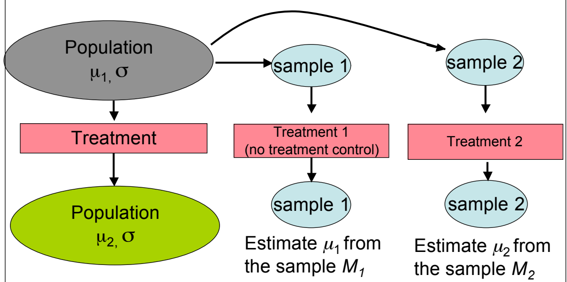
Both values must be estimated from our sample or samples from the population.

Two types of Experimental Designs

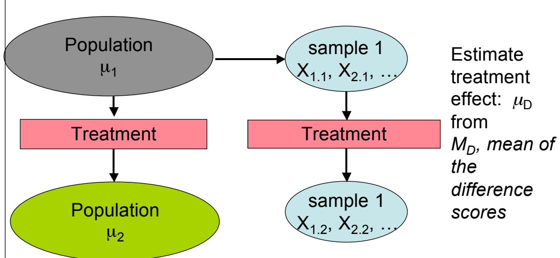


We need estimates of μ_1 and μ_2, and σ

Between Subjects Design (two independent samples)



Within-Subjects Design Related Samples



Examples:

Within groups designs:

pretest-postest (diet, exercise)

repeated measures (taboo memory effect)

Matched groups designs:

yoking studies (executive monkeys)

Difference Scores

Participant	First Score	Second Score	Difference Score
1	$X_{1.1}$	$X_{1.2}$	$X_{1.2} - X_{1.1} = D_1$
2	$X_{2.1}$	$X_{2.2}$	$X_{2.2} - X_{2.1} = D_2$
3	$X_{3.1}$	$X_{3.2}$	$X_{3.2} - X_{3.1} = D_3$
.	.	.	. .
n	$X_{n.1}$	$X_{n.2}$	$X_{n.2} - X_{n.1} = D_n$

M_D = mean difference scores

Difference Scores

Mean difference: $M_D = \Sigma D / n$

$SS_D = \Sigma (D - M_D)^2$ (definition formula)

$SS_D = \Sigma D^2 - (\Sigma D)^2 / N$ (computational formula)

variance of the differences scores

$s_D^2 = SS_D / n-1$

Standard error of the difference scores

$s_{M_D} = \sqrt{s_D^2/n} = s_D / \sqrt{n}$

New formula for *t*

$$t = \frac{\text{sample statistic - population parameter}}{\text{estimated standard error}}$$

$$t = \frac{M_D - \mu_D}{s_{M_D}}$$

Note: μ_D usually equals 0

Hypothesis Testing

1. State H_0, H_1, and choose α
   - H_0: no treatment effect, $\mu_D = 0$
   - H_1: there is treatment effect,
     a) $\mu_D \neq 0$
     b) $\mu_D > 0$
     c) $\mu_D < 0$
2. Determine what type of observation it would take to reject H_0
   a) What is the appropriate test statistic?
      Use the related samples *t* when you do not know the population parameters, and you have 2 closely related samples (e.g., within subjects design).
   b) What is the critical value?
      df = n-1 , n = number of pairs of observations, lookup *t*

Hypothesis Testing

3. Evaluate the sample data
   a) Calculate the differences scores D
   b) Calculate sample M_D, SS_D
   c) Calculate the s_D^2
   d) Calculate the standard error s_{M_D}
   e) Calculate *t*
4. Reach a conclusion

Example:

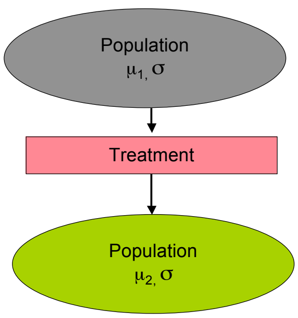
Does hypnotherapy reduce the number of cigarettes smoked. Test at the .05 level.

Participant	Before Hypnotherapy	After Hypnotherapy
1	19	12
2	35	36
3	20	13
4	31	24

Comparing the 2-sample and paired *t* tests

Rework the same problem, but assume the data was from two independent groups of participants.

Confidence Intervals



We have been testing hypotheses concerning population means.

$\mu_1 = \mu_2$

$\mu_1 - \mu_2 = 0$

$\mu_D = 0$

Confidence Intervals

But what are: μ_1 ; μ_2 ; or μ_D ?

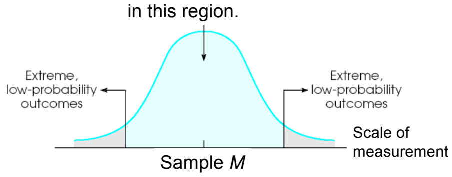
Our best estimates of the population values are obtained from our sample values (e.g, M_1 is our best guess of μ_1).

But these estimates are subject to error, and the size of that error is determined by sample size (law of large numbers).

The central limits theorem tells us that our estimate is determined by the standard error of the mean.

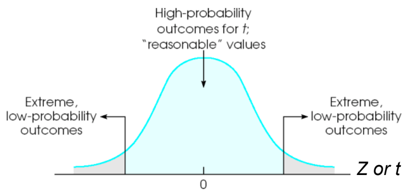
Confidence Intervals

Confidence intervals give a range of scores around the sample mean within which we think the population value lies

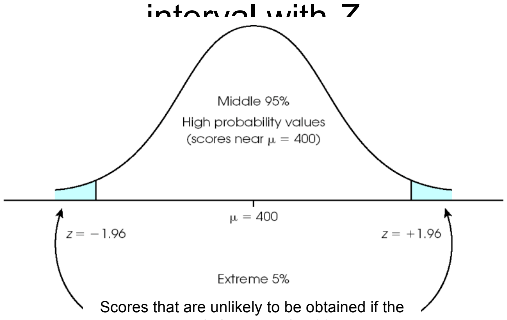


Confidence Intervals

We compute this range by converting our scores to a known probability distribution, like *Z* or *t*.



Example of 95% confidence interval with Z



Confidence Intervals

Steps in constructing confidence intervals:

1. Determine interval size (e.g., 95%)
2. Look up appropriate statistic, using the two tailed value (e.g.,a *t* score).
3. Confidence interval is given by:

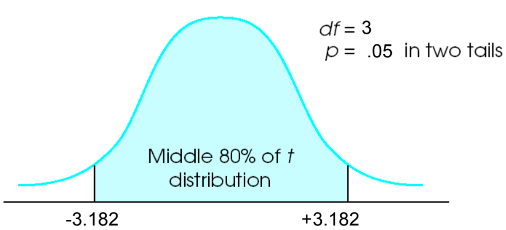
   CI = *M* +/- *t* (standard error)

Confidence Intervals

Example:

Construct the 95% confidence intervals for the smoking study presented earlier.

t values for construction of 95% confidence interval with 3 degrees of freedom



Confidence Intervals

CI = 5 +/- 3.182 (2)

CI = -1.364 to 11.364