

Z scores and the Standard Normal Distribution

- I. Why Z
- II. The Standard Normal Distribution
- III. Calculating Z
- IV. Calculating a score from Z
- V. Z and Standard Scores
- VI. Z and percentiles

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I. Why Z

To compare 1 score to other scores in a distribution.

How did I do on the GRE's?

Determine the effect of a treatment

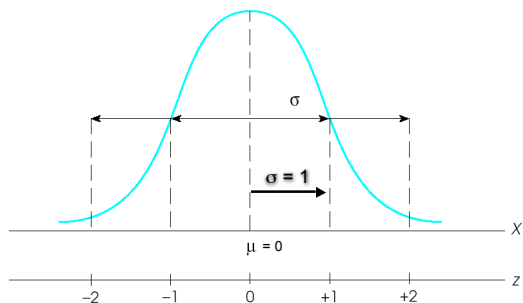
Compared to "normal," what impact did a drug have on my blood pressure?

Compare scores measured on different scales.

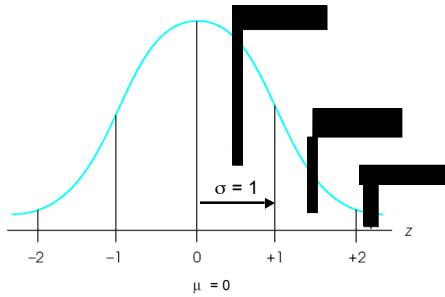
Am I better at math or English?

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II. The Standard Normal Distribution



II. The Standard Normal Distribution



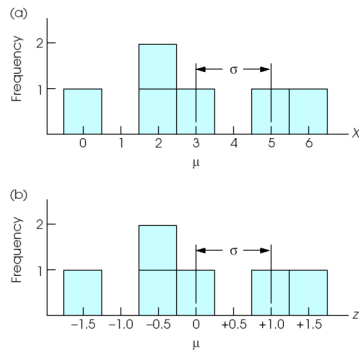
III. Calculating Z

$$Z = \frac{X - \mu}{\sigma}$$

Creates a new distribution with a mean = 0 and a standard deviation equal to 1.

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X	Z
6	1.5
5	1
3	0
2	-0.5
2	-0.5
0	-1.5
$\Sigma = 18$	$\Sigma = 0$
$M = 3$	$M = 0$
$\sigma = 2$	$\sigma = 1$



What does a Z measure

1. The distance of a score from the mean in standard deviation units.
2. We can use the score to compare people across different sets of scores.

Example:	<u>Class 1</u>	<u>Class 2</u>
your grade	37/40	46/50
$\mu =$	28	35
$\sigma =$	6	10
$Z =$	1.5	1.1

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Application: Swimming

100 Back

59.66	Krystle Satrum, F&M
01:01.2	Sarah Bailey, BMC
01:01.5	Katie Cronin, GET
01:01.6	Jen Nolan, URS
01:02.6	Nadine Powell, DC
01:02.7	Emily McGlashon, GET
01:02.8	Allison Simons, WASH
01:02.9	Katie Schmidt, SWAT
01:03.9	Mary Alice Hewitt, F&M
01:03.9	Elizabeth Fouch, GET

200 Back

02:09.4	Krystle Satrum, F&M
02:11.1	Nadine Powell, DC
02:11.2	Katy Cronin, GET
02:13.4	Ally Simons, WASH
02:15.0	Mary Alice Hewitt, F&M
02:15.6	Lisa Hertz, F&M
02:16.0	Elizabeth Fouch, GET
02:16.3	Katie Schmidt, SWAT
02:16.7	Elizabeth Shoemaker, W
02:17.0	Kelly Radimer, BMC

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100 Back			200 Back		
Raw Time	Second	Z-scores	Raw Time	Second	Z-scores
59.66	59.66	-2.22	02:09.4	129.40	-1.94
01:01.2	61.20	-0.91	02:11.1	131.10	-1.25
01:01.5	61.50	-0.66	02:11.2	131.20	-1.21
01:01.6	61.60	-0.57	02:13.4	133.40	-0.31
01:02.6	62.60	0.27	02:15.0	135.00	0.34
01:02.7	62.70	0.36	02:15.6	135.60	0.58
01:02.8	62.80	0.44	02:16.0	136.00	0.74
01:02.9	62.90	0.53	02:16.3	136.30	0.87
01:03.9	63.90	1.38	02:16.7	136.70	1.03
01:03.9	63.90	1.38	02:17.0	137.00	1.15
Mean	62.28		Mean	134.17	
StdPop	1.1787451		StdPop	2.4609865	

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IV. Calculating a score from Z

$$Z = \frac{X - \mu}{\sigma}$$

$$Z \sigma = X - \mu$$

$$Z \sigma + \mu = X$$

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Calculating a score from Z

Example: What would I have to score on a test, with mean = 28, and a standard deviation of 6, to have a Z = 2.0 ?

$$X = Z \sigma + \mu$$

$$X = 2(6) + 28$$

$$X = 40$$

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V. Z and Standard Scores

Z are hard to work with

- positive and negative values
- no one wants to get a zero

To address this, scores are often converted to a standard score that have the properties desired.

$$X_{\text{standard}} = Z \sigma_{\text{stand}} + \mu_{\text{stand}}$$

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Z and Standard Scores

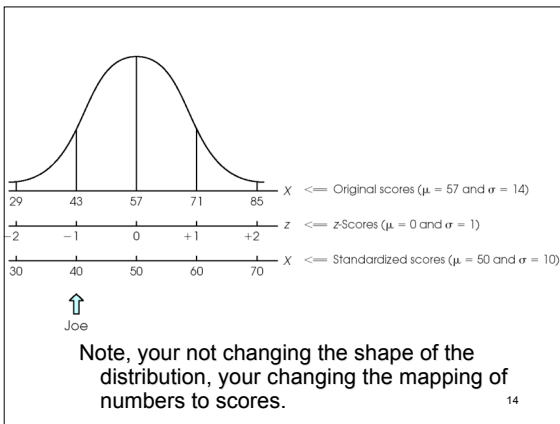
Example: SAT scores
standardized to have a
 $\mu = 500$; and $\sigma = 100$

Raw Score \rightarrow Z score \rightarrow Standard Score

$\mu = 65$ $\mu = 0$ $\mu = 500$
 $\sigma = 10$ $\sigma = 1$ $\sigma = 100$

$X = 70$ $Z = .5$ Stand. Score = 550

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Examples of Standard Scores

SAT
ACT
IQ

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VI. Z and percentiles

The standard unit normal distribution

- Normal curve (bell shaped)
- Mean = 0
- Standard Deviation = 1

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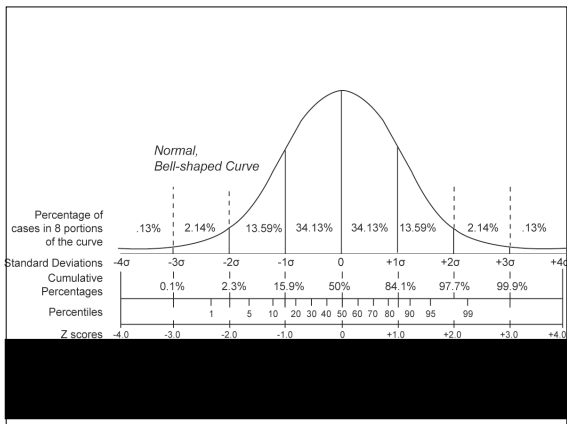
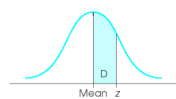
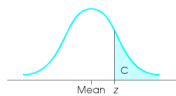
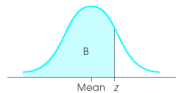


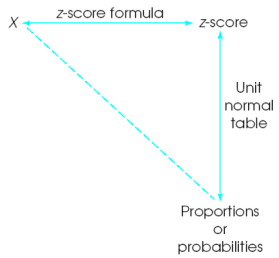
Table of Z (p. 699)

(A) z	(B) Proportion in body	(C) Proportion in tail	(D) Proportion between mean and z
0.00	.5000	.5000	.0000
0.01	.5040	.4960	.0040
0.02	.5080	.4920	.0080
0.03	.5120	.4880	.0120
0.21	.5832	.4168	.0832
0.22	.5871	.4129	.0871
0.23	.5910	.4090	.0910
0.24	.5948	.4052	.0948
0.25	.5987	.4013	.0987
0.26	.6026	.3974	.1026
0.27	.6064	.3936	.1064
0.28	.6103	.3897	.1103
0.29	.6141	.3859	.1141
0.30	.6179	.3821	.1179
0.31	.6217	.3783	.1217
0.32	.6255	.3745	.1255
0.33	.6293	.3707	.1293
0.34	.6331	.3669	.1331



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You can use this table to go from a Z score to a percentile, and from a percentile to a Z.



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Examples

What is the Z score for:

30 percentile

50 percentile

75 percentile

What is the percentile rank for:

Z = 1

Z = -.5

If my IQ is 125, what is my percentile rank?

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