

## Introduction to t

- I. Hypothesis testing with Z, a review
- II. Estimating pop. Standard Deviation
- III. The t distribution & table
- IV. The t formula
- V. Steps
- VI. Examples
- VII. Effect Size

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## Hypothesis testing with Z, a review

1. State  $H_0$ ,  $H_1$ , and choose  $\alpha$
2. Determine what type of observation it would take to reject  $H_0$ 
  - a) What is the appropriate test statistic?
  - b) What is the critical value?
3. Evaluate the sample data
4. Reach a conclusion

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## Hypothesis testing with Z, a review

2. Determine what type of observation it would take to reject  $H_0$ 
    - a) What is the appropriate test statistic?
    - b) What is the critical value?
- With a known  $\mu$ , and  $\sigma$ , we can calculate probabilities with Z

$$Z = \frac{M - \mu}{\sigma_M} \quad \sigma_M = \frac{\sigma}{\sqrt{n}}$$

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## Estimating pop. Standard Deviation

But what if do not know  $\sigma$ , the population standard deviation?

We have to estimate it from our sample:

Remember:

$$SS = \sum (x-M)^2$$

	Population	Sample
Variance:	$\sigma^2 = SS/N$	$s^2 = SS/n-1$

Standard Deviation:	$\sigma = \sqrt{SS/N}$	$s = \sqrt{SS/n-1}$
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## Estimating pop. Standard Deviation

Similarly, estimate the standard error:

$$\sigma_M = \frac{\sigma}{\sqrt{n}} \quad s_M = \frac{s}{\sqrt{n}} = \sqrt{\frac{s^2}{n}}$$

Also, remember your estimates of population values get better with sample size.

Also,  $df = n-1$

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## The t distribution & table

Previously we argued that the distribution of sample means was normal (central limits theorem), so we could use  $Z$  to test hypotheses about those means.

The  $t$  distribution is a probability distribution (similar to  $Z$ ) that takes into account that we are estimating the standard deviation, and this estimate changes with degrees of freedom (sample size). The distribution is normal when  $n$  is large, but flatter than normal for small values of  $n$ .

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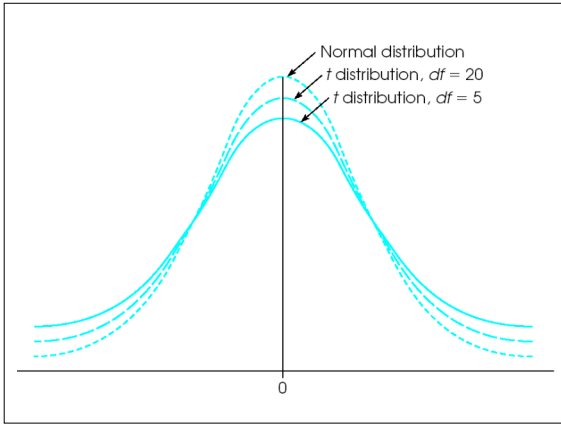
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t table (see p. 703)

df	Proportion in One Tail					
	0.25	0.10	0.05	0.025	0.01	0.005
	Proportion in Two Tails Combined					
	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707

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Hypothesis testing with Z

$$Z = \frac{M - \mu}{\sigma_M} \quad \sigma_M = \frac{\sigma}{\sqrt{n}}$$

Hypothesis testing with t

$$t = \frac{M - \mu}{s_M} \quad s_M = \frac{s}{\sqrt{n}}$$

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## Steps

1. State  $H_0$ ,  $H_1$ , and choose  $\alpha$
2. Determine what type of observation it would take to reject  $H_0$ 
  - a) What is the appropriate test statistic?
  - b) What is the critical value? (calculate  $df$ , lookup  $t$ )
3. Evaluate the sample data
  - a) Calculate sample  $M$  and  $s$
  - b) Calculate the standard error
  - c) Calculate  $t$
4. Reach a conclusion

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## Examples

Are basketball players taller than average?  
Average males in the US at 5'9" (69") tall.

Here is a random sample of basketball players:

6'4", 6'6", 6'2", 6'4"

With alpha equal to .05, test if basketball players as significantly taller than average.

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## Steps

- 1)  $H_0: \mu_1 = \mu_2$   
 $H_1: \mu_1 < \mu_2$   
 $\alpha = .05$
- 2) a) use a  $t$  (you don't know  $\sigma$ )  
b)  $t_{crit}(3) > 2.353$
- 3) Calculate  $M$ ,  $SS$ ,  $s$ ,  $s_M$ ,  $t$
- 4)  $t_{obs} > t_{crit}$ , reject  $H_0$

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## Other examples

These will be done in class as appropriate.

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## Effect Size

What if we find a statistically significant effect, is the effect “meaningful.”

Effect size is a way to quantify the magnitude of a treatment effect.

Two measures:

Cohen’s  $d$

$r^2$

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## Effect Size

Cohen’s  $d$

*Measures how big the effect is by comparing it to the standard deviation.*

Cohen’s  $d = \text{mean difference}/\text{standard dev.}$

In our basketball example:

Cohen’s  $d = 76-69/1.63 = 4.29$

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## Effect Size

Interpreting Cohen's  $d$ :

Magnitude of $d$	Evaluation of Effect Size
$0 < d < 0.2$	Small effect (mean difference less than 0.2 standard deviation)
$0.2 < d < 0.8$	Medium effect (mean difference around 0.5 standard deviation)
$d > 0.8$	Large effect (mean difference greater than 0.8 standard deviation)

Cohen's  $d$  seems arbitrary.

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## Effect Size

$r^2$  as a measure of effect size

$r^2$  measures the proportion of variance in the data that is accounted for by the treatment effect.

total variance in the data = treatment effect  
+ unexplained variance  
(error)

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## Effect Size

$$r^2 = \frac{\text{variance explained by treatment}}{\text{total variance in the data}}$$

Two ways to calculate  $r^2$

1) calculate  $SS_{\text{total}}$ ,  $SS_{\text{treatment}}$ ,  $SS_{\text{error}}$   
 $r^2 = SS_{\text{treatment}} / SS_{\text{total}}$  (the hard way)

2)  $r^2 = f^2 / (f^2 + df)$

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## Effect Size

Basketball example:

$$r^2 = \frac{t^2}{(t^2 + df)}$$
$$= \frac{8.54^2}{(8.54^2 + 3)} = .96$$

That is, 96% of the variance from the population mean can be explained by the fact that my sample is basketball players.

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## Effect Size

Interpreting  $r^2$

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Percentage of Variance Explained,  $r^2$

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$0.01 < r^2 < 0.09$	Small effect
$0.09 < r^2 < 0.25$	Medium effect
$r^2 > 0.25$	Large effect

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