

Edge Orderings, Edge Colourings and Altitude of Cubic Graphs

Kieka Mynhardt*, Alewyn Burger,
Trevor Clark, Bálint Falvai and Nick Henderson
University of Victoria

Abstract

The *altitude* $\alpha(G)$ of a graph G is the largest integer k such that for each linear ordering f of its edges, G has a (simple) path P of length k for which f increases along the edge sequence of P . The altitude of a graph G is bounded above by its chromatic index $\chi'(G)$, and sometimes an edge ordering of G which realizes $\alpha(G)$ corresponds to a $\chi'(G)$ -edge colouring of G . We shall give a necessary and sufficient condition in terms of 4-edge colourings for cubic graphs with girth at least five to have altitude three. Using this result we shall show that some snarks, including all but one of the Blanuša type snarks, have altitude three while others, including the Petersen graph and Isaac's flower snarks, have altitude four. Several open problems relating to altitude and edge colourings will be mentioned.

Keywords: edge-ordering, increasing paths, monotone paths, altitude, snarks