Homomorphism-homogeneous graphs

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Abstract

A graph G is said to be *homogeneous* if any isomorphism between finite induced subgraphs of G extends to an automorphism of G. This is a very strong symmetry condition; and, indeed, the finite and countably infinite homogeneous graphs have been determined by Sheehan, Gardiner, Lachlan and Woodrow.

What if we replace "isomorphism" by "homomorphism" in this definition? (A graph homomorphism is a map of vertices which carries edges to edges.) It turns out that the finite graphs with this property are trivial, but there are many countable graphs, and we do not yet have a complete determination of them.

If we replace "isomorphism" by "monomorphism" (or one-to-one homomorphism), we get a concept for which some general theory can be developed, applying not only to graphs but to arbitrary relational structures. This resembles Fraïssé's theory relating homogeneity to the amalgamation property for finite substructures.

This is joint work with Jarik Nešetřil of Charles University, Prague.