Plane graphs with positive curvature

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Abstract

Let G be a plane graph (finite or infinite) such that (1) G is locally finite and (2) every face of G is bounded by a cycle. Then the combinatorial curvature of G is the function $\Phi(G) : V(G) \longrightarrow R$ such that for any $x \in V(G)$,

$$\Phi(x) = 1 - d(x)/2 + \sum_{x \in F} 1/|F|,$$

where the summation is taken over all facial cycles of G containing x. The curvature interprets the degree of difficulty of tiling the plane at x and it is dual of another curvature introduced by Gromov. Higuchi proved that there is a negative real number μ such that $\Phi(x) < \mu$ if $\phi(x) < 0$ and the positive curvature can be arbitrarily small. We show that if $\Phi(x) \geq 0$ then $\sum_{x \in V(G)} \Phi(x)$ is bounded if and only if there are only finite number x such that $\Phi(x) \neq 0$. Higuchi also conjectured that G is finite if $\Phi(x) > 0$ for all x. Sun and Yu proved this for cubic plane graphs. We completely characterized finite graphs with positive curvatures provide the number of vertices is large.

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