

The Steiner Problem in the hypercube

Tao Jiang ^{*}, Dan Pritikin
Miami University

Abstract

The Steiner problem seeks, for given a set S of vertices in a connected host graph G , a tree of minimum size that contains all of S . Such a tree is called a Steiner tree for S . The Steiner problem has been extensively studied and has important applications in as diverse areas as VLSI-layout and phylogenetic trees.

Here, we consider the Steiner problem in the hypercube. Given a set S of vertices in the n -dimensional cube Q_n , let $L(S)$ denote the size of a Steiner tree for S . Let $f(n, k)$ denote the maximum value of $L(S)$ over all sets S of k vertices. For all k relatively small compared to 2^n , we obtain asymptotically tight bounds on $f(n, k)$. Our upper bound is of an algorithmic nature while the lower bound is probabilistic. We will also briefly discuss the general behavior of the function $L(S)$.