

Some inequalities relating degrees of adjacent vertices to the average degree, in edge-weighted uniform hypergraphs

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Abstract

Seventeen years ago, or thereabouts, R.L.Perry and the first author discovered a small but potent class of inequalities for uniform hypergraphs with no isolated vertices. The nicest of these was: if you take the geometric mean, over the hyperedges, of the geometric mean of the degrees of the vertices on each edge, the result will be no less than the (arithmetic) average degree of the vertices in the hypergraph, with equality if and only if the hypergraph is regular. Here we generalize all of that to the case of uniform hypergraphs with fuzzy, or, more generally, weighted edges, with the "degree" of each vertex being the sum of the weights of the edges that the vertex sits on, with each edge counted according to the multiplicity of the vertex on it. (So, the earlier work is in the special case in which each (hyper)edge gets a weight of 1.) Besides generalizing the earlier results, the fuzzification permits the opening of a new territory for exploration.