Determinants and Rotations

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Abstract

Consider an $n \times n$ matrix $A = [a_{ij}]$ whose n^2 entries are independent variables. The determinant of A is a multilinear polynomial in the a_{ij}'s. For any $\alpha \in \mathbb{Z}^n_{\geq 0}$, let $A[\alpha]$ denote the matrix obtained from A by rotating, for each $i = 1, \ldots, n$, row i to the right α_i units with wrap around. In this paper we prove that, for any vector $\alpha \in \mathbb{Z}^n_{\geq 0}$, the determinant of $A[\alpha]$ is an integer linear combination of the $n!$ determinants in the set

$$\{\det(A[\beta]) : \beta \in \mathbb{Z}^n_n : \beta_i < n - i, \text{ for all } i = 0, \ldots, n-1\}.$$