

IMPROVEMENTS ON THE THEOREM OF DUCHET AND MEYNIEL ON HADWIGER'S CONJECTURE

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Abstract

Probably the best-known among the remaining unsolved problems in graph theory is

Hadwiger's Conjecture: If $\chi(G)$ denotes the chromatic number of graph G , then G has the complete graph $K_{\chi(G)}$ as a minor.

The following would immediately follow from the truth of Hadwiger's Conjecture:

Conjecture: If G has n vertices and if $\alpha(G)$ denotes the independence number of G , then G has $K_{\lceil n/\alpha(G) \rceil}$ as a minor.

In 1982, Duchet and Meyniel proved the following

Theorem: If G has n vertices, then G has $K_{\lceil n/(2\alpha(G)-1) \rceil}$ as a minor.

Until now, no improvement on the Duchet-Meyniel result has been obtained for graphs in general. In this talk, we present the following two results.

Theorem 1: Every graph G on n vertices with $\alpha(G) \geq 2$ and clique number $\omega(G)$ has $K_{\lceil (n+\omega(G))/(2\alpha(G)-1) \rceil}$ as a minor.

Theorem 2: Every graph G on n vertices with $\alpha(G) \geq 3$ has $K_{\lceil n(1+c)/(2\alpha(G)-1) \rceil}$ as a minor, for some constant $c > 0$, c depending on $\alpha(G)$.