

The nonorientable genus of joins of complete graphs with large empty graphs

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Abstract

In 1965, Ringel showed that the nonorientable genus of a complete bipartite graph is $\tilde{g}(K_{m,n}) = \lceil \frac{(m-2)(n-2)}{2} \rceil$. But bipartite graphs have no odd cycles, hence embeddings of bipartite graphs have no triangular faces. One sees then that it is possible to add edges to a complete bipartite graph without increasing the genus. In particular, we prove that if $m \leq n - 1$, then the nonorientable genus of $K_m + \overline{K_n}$ is the same as that of $K_{m,n}$, except for the case when $m = 5$ and $n = 4$. We also present a few conjectures corresponding to alternative ways of adding edges to a complete bipartite graph.