Birth control for giants

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Abstract

We consider a problem of Dimitris Achlioptas. Initially a graph G has no edges. In each round two edges of K_n are generated independently and uniformly at random. We must select one of those edges and add it to G. Our object is to avoid creating a giant component (one with about a constant times n vertices), for as long as possible. This motivates the following problem. Fix any algorithm that determines which edge we select. Let G_m denote the graph after m rounds. Then $G_0, G_1, \ldots$ forms a random graph process, that evolves from the empty graph to a graph with a giant component and, of course, beyond. For a general class of algorithms we analyse this process and show the existence of a phase transition in the size of the largest component. For the standard random graphs, it is well known that this phase transition occurs when about $n/2$ edges are in the graph. For some algorithms we obtain lower bounds of more than $0.8n$ edges for the phase transition. This bound applies to the original Achlioptas problem. For the converse problem of trying to accelerate the birth of the giant, we have upper bounds of less than $0.35n$. This is joint work with Joel Spencer.