Super Connectivity of Line Digraphs

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Abstract

A graph G is super (resp. edge-)connected if every minimum vertex-(resp. edge-) cut isolates a vertex of G. A quite natural problem is that if a connected graph G is super (resp. edge-) connected then how many vertices (resp. edges) must be removed to disconnect G such that every connected component of the resulting graph contains no isolated vertices. This problem results in the concept of the super (resp. edge-) connectivity $\kappa'(G)$ (resp. $\lambda'(G)$), which is defined as the minimum number of vertices (resp. edges) of G whose removal disconnects G such that every connected component of the resulting graph contains no isolated vertices. The super connectivity κ' and the super edge-connectivity λ' are more refined network reliability indices than the connectivity κ and the edge-connectivity λ.

This paper shows that for a connected balanced digraph G and its line digraph L, if G is optimally super edge-connected, then $\kappa'(L) = 2\lambda'(G)$. As consequences, we determine that the super connectivity of de Bruijn digraph $B(d, n)$ and Kautz digraph $K(d, n)$, that is, $\kappa'(B(d, n))$ is equal to $4d - 8$ for $n = 2$ and $d \geq 4$, and to $4d - 4$ for $n \geq 3$ and $d \geq 3$, and that $\kappa'(K(d, n))$ is equal to $4d - 4$ for $d \geq 2$ and $n \geq 2$ except $K(2, 2)$. These results show that $B(d, n)$ and $K(d, n)$ are both super connected for any $d \geq 2$ and $n \geq 1$.

Keywords: Super connectivity, Super edge-connectivity, Line graphs, de Bruijn digraphs, Kautz digraphs

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