PROGRAM : NEWTON

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PURPOSE : To compute the value of a zero (root) of a function $f(x)$.

PLATFORM: Texas Instruments TI 82/83 graphing calculator

DEFINITION: $x = r$ is a *root* or *zero* of a function $f(x)$ means that $f(r) = 0$
The program implements Newton's method to approximate the root of a function $f(x)$.
The derivative is approximated by a centered difference (CDQ):

$$x_{new} = x_{old} - \frac{f(x_{old})}{f'(x_{old})} \approx x_{old} - f(x_{old}) \,/\left(\frac{f(x_{old}+h) - f(x_{old}-h)}{2h}\right)$$

which is applicable to any continuous, differentiable function.
On the TI calculator, the CDQ is available as the nDeriv command.

Input : The function $f(x)$ is stored in $\mathbf{y}_1$ prior to running the program.

Output: the approximate value of the root, stored in A

Note: For the program to successfully determine the value of a root of $f(x)$ make sure that:
- the function actually has a root by graphing $f(x)$
- and that both the function and its derivative exist on the interval of interest.

Particularly, vertical slopes can be a problem.

PROGRAM NEWTON	Command Locations
`:Prompt X`	Prompt: prgm I/O
`:X-y1(X)/nDeriv(y1(X),X,X)→A`	→: Sto
`:While Abs(X-A)>1EE-8`	While: Prgm Ctl
`:A→X`	
`:X-y1(X)/nDeriv(y1(X),X,X)→A`	nDeriv: Math 8
`:Disp A`	Disp: Prgm Ctl
`:End`	End: Prgm Ctl
`:Stop`	Stop: Prgm Ctl

EXAMPLE: To solve the equation $3^x = 5x$ for x, we need to determine the root of the function $f(x) = 3^x - 5x$. (Answers: start with $x=0$: 0.26866911 and
start with $x=2$: 2.17027659. The equation has two solutions.

EXAMPLE: To solve the equation $x^4 - 4x^2 = x - 2$ for x,
we need to determine the root of the function $f(x) = x^4 - 4x^2 - x + 2$,
(check graphically that there are four distinct real roots!).
start with $x=-2$: -1.65121076
start with $x=-1$: -1 (bingo!)
start with $x=0$: 2
start with $x=2$: 0.61803399. The equation has four solutions.