Let me count the ways . . . .

A Problem Solving Workshop

Expanding Your Horizons

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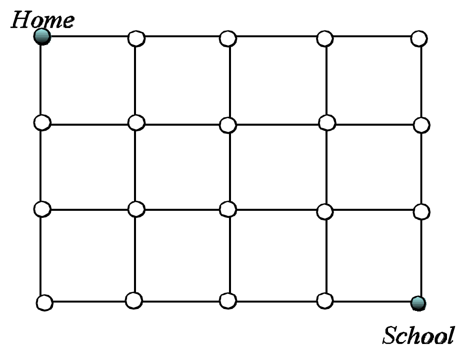
1. Counting paths in a rectangular grid.

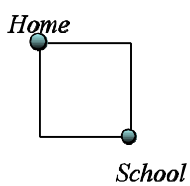
Keith's school is located four blocks over and three blocks down from his home.



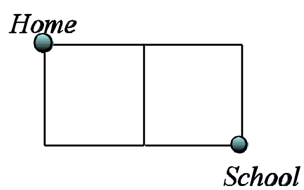
He wants to know **how many different routes he can ride his bike from home to school**, if <u>no backtracking</u> is allowed - he can only go *over* or *down* in the grid.

a. Sketch one possible route Keith can take from home to school in the above grid

b. You see that there are many different routes possible. Counting them all is no small matter! Let's first *simplify* the situation, and count the number of possible routes:

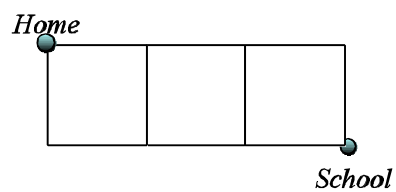


1 block over,
3 blocks over,
and 1 block down?
__________ routes.

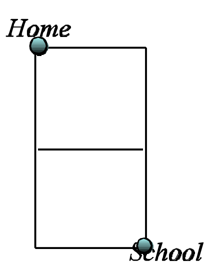


2 blocks over,

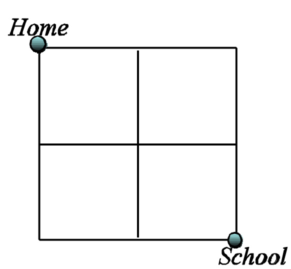
and 1 block down?
__________ routes.



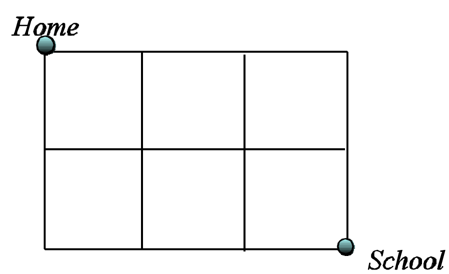
and 1 block down?
__________ routes.



1 block over,
and 2 blocks down?
__________ routes.

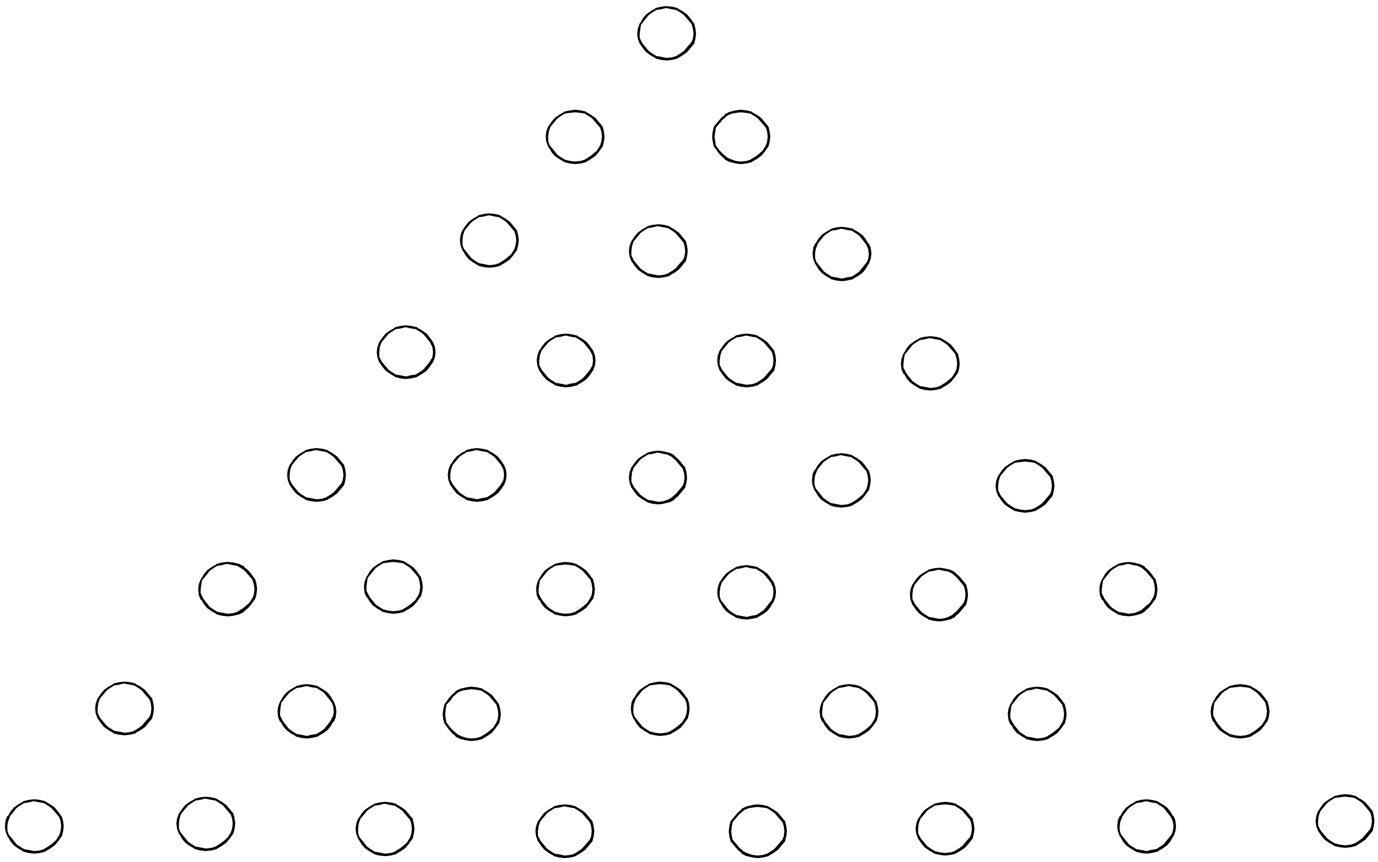


2 blocks over,
and two blocks down?
__________ routes.



Three blocks over
and two blocks down?
__________ routes.

c. Next, let's look for a *pattern* in the numbers above: transfer your results on the grid, and fill out the triangle known as *Pascal's triangle* on the next page:



Pascal's Triangle

Do you see a pattern in the numbers in the triangle?   Describe the pattern

Each number in the triangle is the __________ of the numbers ________________ it.

d. Fill out the triangle using this pattern

e. Use the completed triangle to answer the original question:
If Keith's school is located four blocks over and three blocks down from his home, how many different routes he can ride his bike from home to school?

Answer: __________

2. Counting amounts of money.

In this problem, you need four distinct coins: a *penny*, a *nickel*, a *dime* and a *quarter*.

Question: **how many different *amounts* can be formed using four different coins**.

For instance - you can select the *penny* and the *dime*. Amount: _______ cents.
or you can select *none* of the four coins. Amount: _______ cents.
or you can select *all* of the four coins. Amount: _______ cents.

There are many more possible amounts using *none*, *some* or *all* of the four coins.

To answer the question, we'll start by *simplifying* the problem.

- Suppose you have *no coins at all* . How many different amounts can you form?
___

- Suppose you have *just one coin.* (say, the penny).
How many different amounts can you form
using *no coins*? ___ Using *one coin*? ___

- Suppose you have *two coins.* (say, the penny and the nickel).
How many different amounts can you form
using *no coins*? ___ Using *one coin*? ___ Using *two coins*? ___

- Suppose you have *three coins.* (say, the penny, the nickel and the dime).
How many different amounts can you form
using *no coins*? ___ Using *one coin*? ___ Using *two coins*? ___ Using *three coins*? ___

- Suppose you have *four coins.* (say, the penny, the nickel, the dime and the quarter).
How many different amounts can you form
using *no coins*? ___ Using *one coin*? ___ Using *two coins*? ___ Using *three coins*? ___ Using *four coins*? ___.

Adding up the numbers on each line, we can make table of results:

Number of Coins	Different Amounts Possible
0	
1	
2	
3	
4	

From the pattern in the table, can you predict **how many different amounts can be formed using *five* different coins?** __________ .

3. The problem of Collatz

Sometimes, a *sequence* or *list* of numbers ends up in a fixed value, regardless of your choice for a starting number.

An example of such a problem is the following sequence:

Starting with *any* counting number, obtain the following number by:

- dividing by 2 if the previous number is even
- multiply by 3 and add 1 if the previous number is odd.

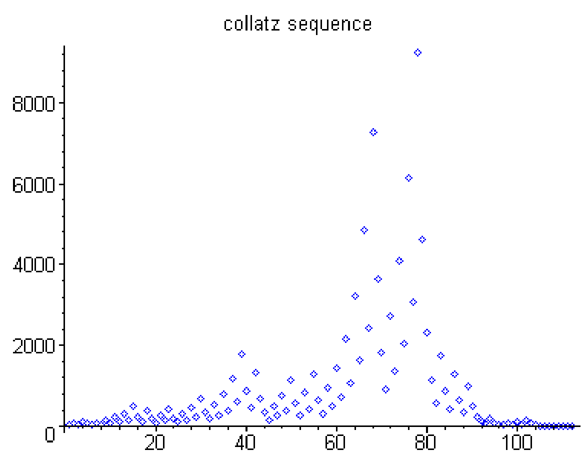
For instance: starting (arbitrarily) with 5, gives: $5 \to 16 \to 8 \to 4 \to 2 \to 1$

Starting at 6, you get: $6 \to 3 \to 10 \to 5 \to$ .. (see above, you've been there!) .. $\to 1$

Try your own number:

example 1	5	16	8	4	2	1										
example 2	6	3	10	5	16	8	4	2	1							
your #																
your #																
your #																

(careful .... as the picture shows, some starting values (say, 27) produce huge sequences!)



The famous problem of the German mathematician Georg Collatz is:

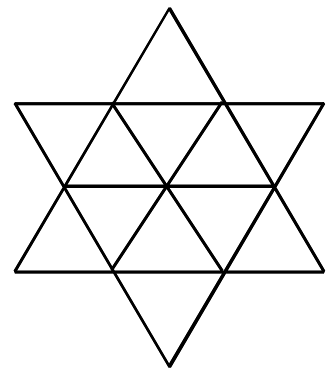
Does the above rule, for *any* starting number, produce a sequence that will end up in the value 1 ?

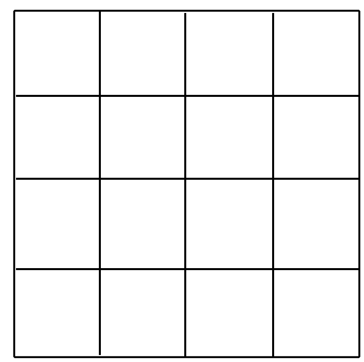
Nobody knows ..... yet .....

This is what's called an *open* problem - something for mathematicians to work on!

You may work the following problems at home.
(You'll find the answers on the website http://www.mtsu.edu/~zijlstra)

4. a. How many different triangles can you find in this star?





b. Count the total number of squares in the grid:

c. Darken four of the squares in the grid so that *none of the darkened squares is on the same horizontal, vertical or diagonal line as another darkened square*.